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Chapter 24

SINGLE-PHASE INDUCTION MOTORS: STEADY STATE

24.1 INTRODUCTION

Steady state performance report in general on the no-load and on-load torque, efficiency and power factor, breakdown torque, locked-rotor torque, and torque and current versus speed.

In Chapter 23, we already introduced a quite general equivalent circuit for steady state (Figure 23.8) with basically only two unknowns-the forward and backward current components in the main windings with space and time harmonics neglected.

This circuit portrays steady state performance rather well for a basic assessment. On the other hand, the d-q model/cross field model (see Chapter 23) may alternatively be used for the scope with $d/dt = j\omega_1$.

The cross field model [1, 2] has gained some popularity especially for T-L tapped and generally tapped winding multi-speed capacitor motors. [3,4] We prefer here the symmetrical component model (Figure 23.18) for the split phase and capacitor IMs as it seems more intuitive.

To start with, the auxiliary phase is open and a genuine single phase winding IM steady state performance is investigated.

Then, we move on to the capacitor motor to investigate both starting and running steady state performance.

Further on, the same investigation is performed on the split phase IM (which has an additional resistance in the auxiliary winding).

Then the general tapped winding capacitor motor symmetrical component model and performance are treated in some detail.

Steady state modelling for space and time harmonics is given special attention.

Some numerical examples are meant to give a consolidated feeling of magnitudes.

24.2. STEADY STATE PERFORMANCE WITH OPEN AUXILIARY WINDING

The auxiliary winding is open, in capacitor start or in split-phase IMs, after the starting process is over (Figure 24.1).

The auxiliary winding may be kept open intentionally for testing. So this particular connection is of practical interest.

According to (23.21)-(23.22) the direct and inverse main winding current components I_{m+} and I_{m-} are

$$\underline{I}_{m+,-} = \frac{1}{2}(\underline{I}_m \mp j\underline{I}_a) = \frac{1}{2}\underline{I}_m \quad (24.1)$$

The equivalent circuit in Figure 23.18 gets simplified as $\underline{Z}_a = \infty$ (Figure 24.2)

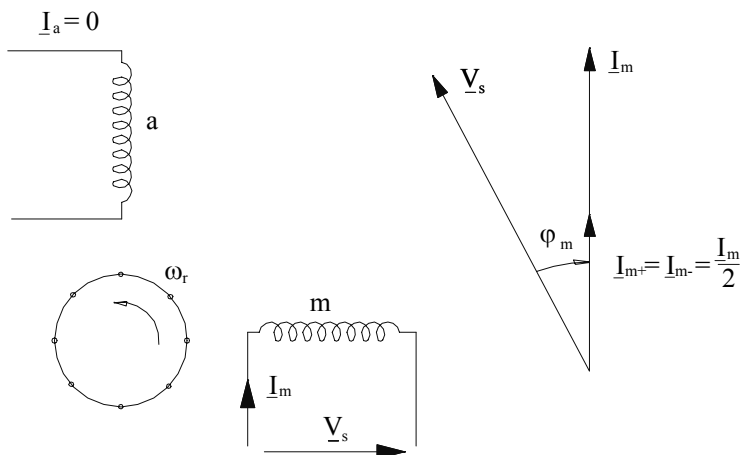


Figure 24.1 The case of open auxiliary winding: a.) schematics, b.) phasor diagram

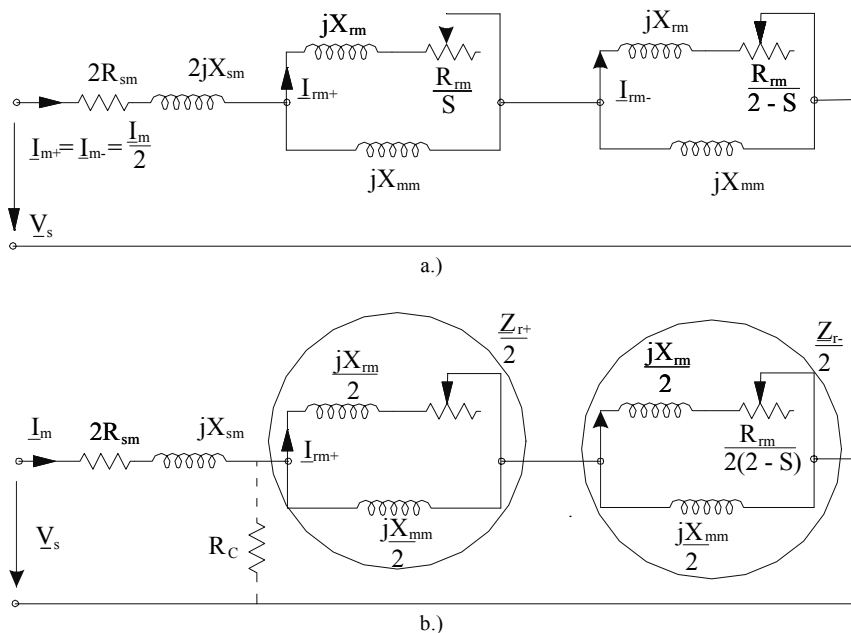


Figure 24.2 Equivalent circuit with open auxiliary phase: a.) with forward current $\underline{I}_{m+} = \underline{I}_{m-} = \underline{I}_m/2$; b.) with total current $\underline{I}_m$

As the two current components I_{m+} and I_{m-} are equal to each other, the torque expression T_e (23.18) becomes

$$T_e = 2 \frac{R_r \cdot p_l}{\omega_1} \left(\frac{I_{rm+}^2}{S} - \frac{I_{rm-}^2}{2-S} \right) \quad (24.2)$$

with

$$I_m = \frac{\frac{V_s}{2}}{R_{sm} + jX_m + \left(\frac{Z_{r+}}{2} \right) + \left(\frac{Z_{r-}}{2} \right)} \quad (24.3)$$

$$\frac{Z_{r+}}{2} = \frac{j \frac{X_{mm}}{2} \left(\frac{R_{rm}}{2S} + j \frac{X_{rm}}{2} \right)}{\frac{R_{rm}}{2S} + j \frac{(X_{mm} + X_{rm})}{2}}$$

$$\frac{Z_{r-}}{2} = \frac{j \frac{X_{mm}}{2} \left(\frac{R_{rm}}{2(2-S)} + j \frac{X_{rm}}{2} \right)}{\frac{R_{rm}}{2(2-S)} + j \frac{(X_{mm} + X_{rm})}{2}} \quad (24.4)$$

It is evident that for $S = 1$ (standstill), though the current I_m is maximum, I_{ms}

$$I_{ms} \approx \frac{V_s}{\sqrt{(R_{sm} + R_{rm})^2 + (X_{sm} + X_{rm})^2}} \quad (24.5)$$

The total torque (24.2) is zero as $S = 2-S = 1$.

The torque versus slip curve is shown on [Figure 24.3](#).

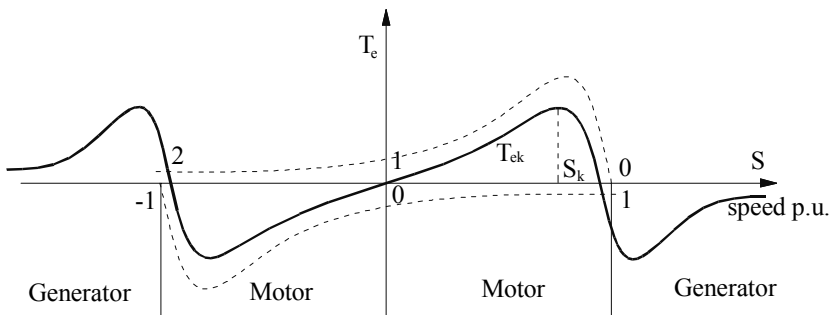


Figure24.3 Torque /speed curve for open auxiliary winding (single phase winding operation)

At zero slip ($S = 0$) the torque is already negative as the direct torque is zero and the inverse torque is negative. The machine exhibits only motoring and generating modes.

The peak torque value T_{ek} and its corresponding slip (speed) S_K are mainly dependent on stator and rotor resistances R_{sm} , R_{rm} and leakage reactances X_{sm} , X_{rm} .

As in reality, this configuration works for split-phase and capacitor-start IMs, the steady state performance of interest relate to slip values $S \leq S_K$.

The loss breakdown in the machine is shown on [Figure 24.4](#).

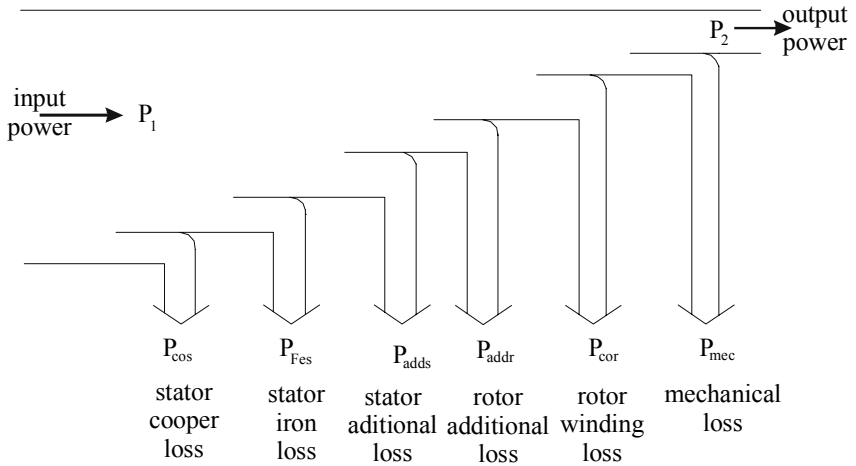


Figure 24.4 Loss breakdown in the single phase IM

Winding losses p_{cos} , p_{cor} occur both in the stator and in the rotor. Core losses occur mainly in the stator, p_{Fes} .

Additional losses occur in iron and windings and both in the stator and rotor P_{adds} , P_{addr} .

The additional losses are similar to those occurring in the three phase IM. They occur on the rotor and stator surface and as tooth-flux pulsation in iron, in the rotor cage and in the rotor laminations, due to transverse interbar currents between the rotor bars.

The efficiency η is, thus,

$$\eta = \frac{P_2}{P_1} = \frac{P_1 - \sum p}{P_1} \quad (24.6)$$

$$\sum p = p_{cos} + p_{Fes} + p_{cor} + p_{adds} + p_{addr} + p_{mec} \quad (24.7)$$

A precise assessment by computation of efficiency (of losses, in fact) constitutes a formidable task due to complexity of the problem caused by magnetic saturation, slot openings, skin effect in the rotor cage, transverse (interbar) rotor currents, etc.

In a first approximation, only the additional losses $p_{add} = p_{adds} + p_{addr}$ and the mechanical losses p_{mec} may be assigned a percentage value of rated output

power. Also, the rotor skin effect may be neglected in a first approximation and the interbar resistance may be lumped into the rotor resistance.

Moreover, the fundamental core loss may be calculated by simply “planting” a core resistance R_c in parallel, as shown in [Figure 24.2b](#).

A numerical example follows.

Example 24.1.

Let us consider a permanent capacitor ($C = 8 \mu\text{F}$) 6 pole IM fed at 230 V, 50 Hz, whose rated speed $n_n = 940$ rpm. The main and auxiliary winding and rotor parameters are: $R_{sm} = 34 \Omega$, $R_{sa} = 150 \Omega$, $a = 1.73$, $X_{sm} = 35.9 \Omega$, $X_{sa} = a^2 X_{sm}$, $X_{rm} = 29.32 \Omega$, $R_{rm} = 23.25 \Omega$, $X_{mm} = 249 \Omega$.

The core loss is $p_{Fes} = 20$ W and the mechanical loss $p_{mec} = 20$ W; the additional losses $p_{add} = 5$ W.

With the auxiliary phase open, calculate

- The main winding current and power factor at $n = 940$ rpm
- The electromagnetic torque at $n = 940$ rpm
- The mechanical power, input power, total losses and efficiency at 940 rpm
- The current and torque at zero slip ($S = 0$)

Solution

- The no load ideal speed n_1 is

$$n_1 = \frac{f_1}{p_1} = \frac{50}{3} = 16.66\text{rps} = 1000\text{rpm}$$

The value of rated slip S_n is

$$s_n = \frac{n_1 - n_n}{n_1} = \frac{1000 - 940}{1000} = 0.06$$

$$2 - S_n = 2 - 0.06 = 1.94!$$

The direct and inverse component rotor impedances $Z_{r+}/2$, $Z_{r-}/2$ (24.3)-(24.4) are

$$\frac{Z_{r+}}{2} = \frac{1}{2} \frac{j24.9(23.25/0.06 + j29.32)}{23.25/0.06 + j(29.32 + 249)} \approx 52.7 + j87$$

$$\frac{Z_{r-}}{2} = \frac{1}{2} \frac{j249(23.25/1.94 + j29.32)}{23.25/1.94 + j(29.32 + 249)} \approx 4.8 + j13.5$$

The total impedance at $S = 0.06$ in the circuit of [Figure 24.2 b](#)) $\underline{Z}_{11}$ is

$$\underline{Z}_{11} = R_{sm} + jX_{sm} + \frac{\underline{Z}_{r+}}{2} + \frac{\underline{Z}_{r-}}{2} = 92.8 + j137$$

Now the RMS stator current I_m , (24.3), is

$$|I_m| = \left| \frac{V_s}{Z_{11}} \right| = \frac{230}{165.19} \approx 1.392A$$

The power factor $\cos\phi_{11}$ is

$$\cos\phi_{11} = \frac{\text{Re al}(Z_{11})}{|Z_{11}|} = \frac{92.8}{165.19} \approx 0.5618 !$$

b) The torque T_e is

$$T_e = \frac{P_l}{\omega_l} I_m^2 [\text{Re}(Z_{r+}) - \text{Re}(Z_{r-})] = \frac{3}{314} \cdot 1.392^2 [52.7 - 4.8] = 0.8867 \text{Nm} \quad (24.8)$$

Notice that the inverse component torque is less than 10% of direct torque component.

c) The iron and additional losses have been neglected so far. They should have occurred in the equivalent circuit but they did not.

So,

$$\begin{aligned} p_{\cos} + p_{\text{cor}} &= P_l - T_e \cdot 2\pi n = I_m^2 \text{Re}[Z_{11}] - T_e \cdot 2\pi n_l = \\ &= 1.392^2 \cdot 92.8 - 0.8867 \cdot 2\pi \cdot 16.6 = 87.1 \text{W} \end{aligned}$$

The mechanical power P_{mec} is

$$P_{\text{mec}} = T_e \cdot 2\pi n - p_{\text{mec}} = 0.8867 \cdot 2\pi \cdot 16.666(1 - 0.06) - 20 = 85.20 \text{W}$$

If we now add the core and additional losses to the input power, the efficiency η is

$$\eta = \frac{85.2}{87.1 + 85.2 + 2 + 20 + 5} = 0.4275$$

The input power P_l is

$$P_l = \frac{P_{\text{mec}}}{\eta} = \frac{85.2}{0.4975} = 199.30 \text{W}$$

The power factor, including core and mechanical losses, is

$$\cos\phi = P_l / (V_s I_m) = 199.30 / (230 \cdot 1.392) = 0.6225 !$$

d) The current at zero slip is approximately

$$\begin{aligned} (I_m)_{s=0} &\approx \left| \frac{\frac{V_s}{s}}{R_{sm} + j \left(X_{sm} + \frac{X_{mm}}{2} \right) + \frac{(Z_{r-})_{s=0}}{2}} \right| \approx \left| \frac{230}{34 + j(35.9 + 249) + 4.8 + j13.5} \right| \\ &= 0.764 \text{A} \end{aligned}$$

Only the inverse torque is not zero

$$(T_e)_{s=0} = -(I_m)_{s=0}^2 \operatorname{Re} \frac{(Z_{r-})_{s=0}}{2} \cdot \frac{P_1}{\omega_1} \approx -0.764^2 \cdot 4.8 \cdot \frac{3}{314} = -0.0268 \text{ Nm}$$

Negative torque means that the machine has to be shaft-driven to maintain the zero slip conditions.

In fact, the negative mechanical power equals half the rotor winding losses as $S_b = 2$ for $S = 0$.

So the rotor winding loss $(p_{\text{cor}})_{s=0}$ is

$$(p_{\text{cor}})_{s=0} = 2 \times (T_e)_{s=0} \cdot 2\pi n_1 = 2 \times 0.0268 \times 2\pi \cdot 16.6 = 5.604 \text{ W}$$

The total input power $(P_1)_{s=0}$ is

$$P_{10} = (R_{\text{sm}} + \operatorname{Re}(Z_b)_{s=0}) (I_m)_{s=0}^2 \approx (34 + 4.8) \cdot 0.764^2 = 22.647 \text{ W}$$

The stator winding losses $p_{\text{cos}} = 34 \times 0.764^2 = 19.845 \text{ W}$

Consequently half of rotor winding losses at $S = 0$ (2.8W) are supplied from the power source while the other half is supplied from the shaft power.

The frequency of rotor currents at $S = 0$ is $f_{2b} = S_b f_1 = 2f_1$.

The steady state performance of IM at $S = 0$ with an open auxiliary phase is to be used for loss segregation and some parameters estimation.

24.3 THE SPLIT PHASE AND THE CAPACITOR IM: CURRENTS AND TORQUE

For the capacitor IM the equivalent circuit of [Figure 23.18](#) remains as it is with

$$Z_a = -j / \omega_1 C \quad (24.9)$$

as shown in [Figure 24.5](#).

For the split phase IM, the same circuit is valid but with $C = \infty$ and $R_{sa}/a^2 - R_{sm} > 0$.

Also for a capacitor motor with a permanent-capacitor in general: $R_{sa}a^2 - R_{sm} = 0$; $X_{sa}a^2 - X_{sm} = 0$; that is, it uses about the same copper weight in both, main and auxiliary windings.

Notice that all variables are reduced to the main winding (see the two voltage components). This means that the main and auxiliary actual currents I_m and I_a are now

$$I_m = I_{m+} + I_{m-}; I_a = j[I_{m+} - I_{m-}] / a \quad (24.10)$$

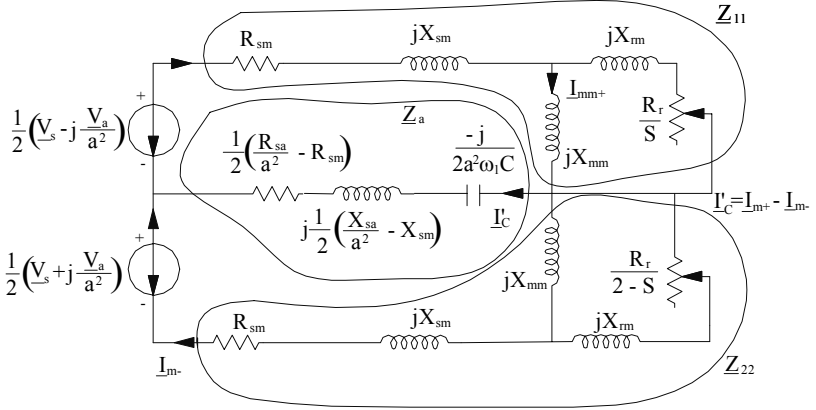


Figure 24.5 Equivalent circuit for the capacitor IM ($V_a=V_s$) in the motor mode

Let us denote

$$\underline{Z}_+ = R_{sm} + jX_{sm} + \frac{jX_{mm}(jX_{rm} + R_{rm}/S)}{R_{rm}/S + j(X_{mm} + X_{rm})} \quad (24.11)$$

$$\underline{Z}_a^m = \Delta R_{sm} + j\Delta X_{sm} - \frac{j}{2a^2} X_c : X_c = \frac{1}{\omega_l C} \quad (24.12)$$

$$\Delta R_{sm} = \frac{1}{2}(R_{sa}/a^2 - R_{sm}); \Delta X_{sm} = \frac{1}{2}(X_{sa}/a^2 - X_{sm}) \quad (24.13)$$

$$\underline{Z}_- = R_{sm} + jX_{sm} + \frac{jX_{mm}(jX_{rm} + R_{rm}/(2-S))}{R_{rm}/(2-S) + j(X_{rm} + X_{mm})} \quad (24.14)$$

The solution of the equivalent circuit in Figure 23.19, with (24.11)-(24.14), leads to matrix equations

$$\begin{bmatrix} \frac{\underline{V}_s(1-j/a)}{2} \\ \frac{\underline{V}_s(1+j/a)}{2} \end{bmatrix} = \begin{bmatrix} \underline{Z}_+ + \underline{Z}_a^m & -\underline{Z}_a^m \\ -\underline{Z}_a^m & \underline{Z}_- + \underline{Z}_a^m \end{bmatrix} \cdot \begin{bmatrix} \underline{I}_{m+} \\ \underline{I}_{m-} \end{bmatrix} \quad (24.15)$$

with the straightforward solution

$$\underline{I}_{m+} = \frac{\underline{V}_s}{2} \frac{(1-j/a)(\underline{Z}_- + 2\underline{Z}_a^m)}{\underline{Z}_+ \cdot \underline{Z}_- + \underline{Z}_a^m(\underline{Z}_+ + \underline{Z}_-)} \quad (24.16)$$

$$\underline{I}_{m-} = \frac{\underline{V}_s}{2} \frac{(1+j/a)(\underline{Z}_+ + 2\underline{Z}_a^m)}{\underline{Z}_+ \cdot \underline{Z}_- + \underline{Z}_a^m(\underline{Z}_+ + \underline{Z}_-)} \quad (24.17)$$

The electromagnetic torque components T_{e+} and T_{e-} retain the expressions (24.8) under the form

$$T_{e+} = \frac{2p_1}{\omega_1} I_{m+}^2 [\operatorname{Re}(\underline{Z}_+) - R_{sm}] \quad (24.18)$$

$$T_{e-} = \frac{2p_1}{\omega_1} I_{m-}^2 [\operatorname{Re}(\underline{Z}_-) - R_{sm}] \quad (24.19)$$

The factor 2 in (24.18)-(24.19) is due to the fact that the equivalent circuit on Figure 23.19 is per phase and it refers to a two phase winding model.

Apparently, the steady state performance problem is solved once we have the rather handy Equations. (24.10)-(24.18) at our disposal.

We now take a numerical example to get a feeling of magnitudes.

Example 24.2. Capacitor motor case

Let us consider again the capacitor motor from example 24.1.; this time with the auxiliary winding on and R_{sa} changed to $R_{sa} = a^2 R_{sm}$, $C' = 4\mu F_0$ and calculate

- The starting current and torque with $C = 8 \mu F$
- For rated slip $S_n = 0.06$: supply current and torque
- The main phase I_m and auxiliary phase current I_a and the power factor at rated speed ($S_n = 0.06$)
- The phasor diagram

Solution

- For $S = 1$, from (24.10)-(24.13),

$$\begin{aligned} (\underline{Z}_+)_{S=1} &= (\underline{Z}_-)_{S=1} = \underline{Z}_{sc} \approx R_{sm} + R_{rm} + j(X_{sm} + X_{rm}) \\ &= (34 + 23.25) + j(35.9 + 29.32) \\ &= 57.5 + j65.22 \end{aligned}$$

Note that $\Delta R_{sm} = \frac{R_{sa}}{a^2} - R_{sm} = 0$ and $\Delta X_{sm} = \frac{X_{sa}}{a^2} - X_{sm} = 0$ and thus

$$\underline{Z}_a^m = \frac{-jX_c}{2a^2} = \frac{-j}{2 \cdot 1.73^2 \cdot 2\pi \cdot 50 \times 8 \cdot 10^{-6}} \approx -j66.34$$

From (24.16)-(24.17)

$$\begin{aligned} (I_{m+})_{S=1} &= \frac{230}{2} \frac{[(1 - j/1.73)(57.5 + j65.22) - 2j66.7]}{(57.5 + j65.22)^2 - j66.7 \cdot (57.5 + j65.22) \times 2} \\ &= 1.444 - j1.47 \end{aligned}$$

$$(\underline{I}_{m-})_{s=1} = \frac{230}{2} \frac{[(1 + j/1.73)(57.5 + j65.22) - 2j \cdot 66.7]}{(57.5 + j65.22)^2 - j66.7 \cdot (57.5 + j65.22) \times 2} \\ = 0.3 - j0.5$$

Consequently, the direct and inverse torques are ((24.17)-(24.18))

$$(T_{e+})_{s=1} = \frac{2 \times 3}{314} \cdot 2.06^2 (57.5 - 34) = 1.9066 \text{ Nm}$$

$$(T_{e-})_{s=1} = \frac{-2 \times 3}{314} \cdot 0.583^2 (57.5 - 34) = -0.1526 \text{ Nm}$$

The inverse torque T_{e-} is much smaller than the direct torque T_{e+} . This indicates that we are close to symmetrization through adequate a and C .

The main and auxiliary winding currents $\underline{I}_m$ and $\underline{I}_a$ are (24.10)

$$\underline{I}_m = \underline{I}_{m+} + \underline{I}_{m-} = 1.444 - j1.47 + 0.30 - j0.50 \approx 1.744 - j1.97$$

$$\underline{I}_a = \frac{j(\underline{I}_{m+} - \underline{I}_{m-})}{a} = \frac{j(1.44 - j1.47 - 0.3 + j0.5)}{1.73} = j0.6613 + 0.56$$

$$\underline{I}_s = \underline{I}_m + \underline{I}_a = 1.744 - j1.97 + j0.6693 + 0.56 = 2.304 - j1.308$$

At the start, the source current is lagging the source voltage (which is a real variable here) despite the influence of the capacitance.

The capacitance voltage,

$$(V_c)_{s=1} = \frac{(\underline{I}_a)_{s=1}}{\omega C} = \frac{0.866}{314 \cdot 8 \cdot 10^{-8}} = 344.75 \text{ V}$$

b) For rated speed (slip) $-S_n = 0.06$, the same computation routine is followed. As the capacitance is reduced twice

$$\underline{Z}_a^m = -j133.4$$

$$(\underline{Z}_+)_{s=0.06} = R_{sm} + jX_{sm} + \frac{jX_{mm} \left(\frac{R_{rm}}{S} + jX_{rm} \right)}{\frac{R_{rm}}{S} + j(X_{mm} + X_{rm})} \\ = 34 + j23.25 + \frac{j249 \left(\frac{23.25}{0.06} + j29.32 \right)}{\frac{23.25}{0.06} + j(249 + 29.32)} = 139.4 + j197.75$$

$$(\underline{Z}_{-})_{s=0.06} = 340 + j23.25 + \frac{j249\left(\frac{23.25}{1.94} + j29.32\right)}{\frac{23.25}{1.94} + j(249 + 29.32)} = 46.6 + j50.25\Omega$$

$$(\underline{I}_{m+})_{s_n=0.06} = \frac{230}{2} \frac{[(1 - j/1.73)(46.6 + j50.2) - 2j133.4]}{(139.4 + j197)(46.6 + j50.25) - j133.4 \cdot (139.4 + 46.66 + j(197 + 50.25))} \\ = 0.525 - j0.794$$

$$(\underline{I}_{m-})_{s_n=0.06} = \frac{230}{2} \frac{[(1 + j/1.73)(139 + j197.5) - 2j133.4]}{(139.4 + j197)(46.6 + j50.25) - j133.4 \cdot (139.4 + 46.66 + j(197 + 50.25))} \\ = 0.1016 + j0.0134$$

$$(T_{e+})_{s=1} = \frac{2 \times 3}{314} \cdot (0.9518)^2 (139.4 - 34) = 1.8245 \text{ Nm}$$

$$(T_{e-})_{s=1} = -\frac{2 \times 3}{314} \cdot (0.1025)^2 (46.6 - 34) = -2.53 \times 10^{-3} \text{ Nm}$$

Both the inverse current and torque components are very small. This is a good sign that the machine is almost symmetric. Perfect symmetry is obtained for $\underline{I}_{m-} = 0$.

The main and auxiliary winding currents are

$$\underline{I}_m = \underline{I}_{m+} + \underline{I}_{m-} = 0.525 - j0.794 + 0.1016 + j0.0134 = 0.6266 - j0.7806 \\ \underline{I}_a = j \frac{(\underline{I}_{m+} - \underline{I}_{m-})}{a} = \frac{j(0.525 - j0.794 - 0.1016 - j0.0134)}{1.73} = j0.274 + 0.466$$

The phase shift between the two currents is almost 90° .

Now the source current $(\underline{I}_s)_{s=0.06}$ is

$$(\underline{I}_s)_{s=0.06} = 0.6266 - j0.7806 + j0.274 + 0.466 \\ = 1.0926 - j0.5066$$

The power factor of the motor is

$$\cos \varphi_s = \frac{\text{Re}(\underline{I}_s)}{|\underline{I}_s|} = \frac{1.0926}{1.2043} = 0.9072$$

The power factor is good. The $4\text{ }\mu\text{F}$ capacitance has brought the motor close to symmetry and providing for good power factor.

The speed torque curve may be calculated, for steady state, point by point as illustrated above. (Figure 24.6)

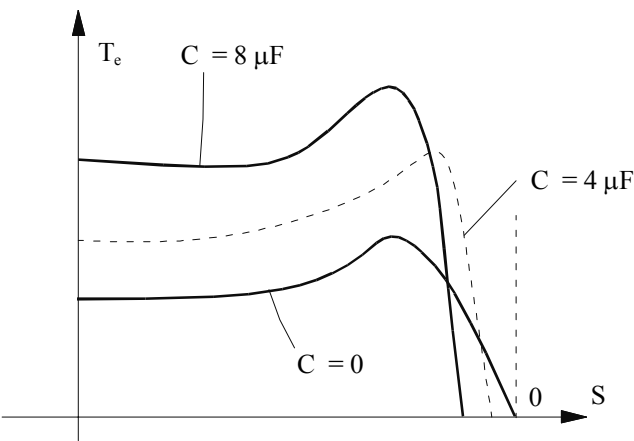


Figure 24.6 Torque speed curve of capacitor motors from example 24.2 (qualitative)

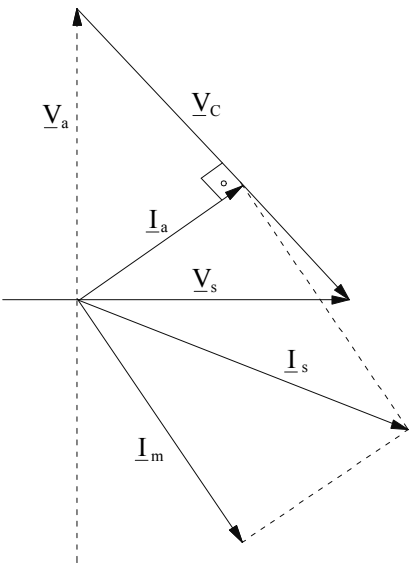


Figure 24.7 Phasor diagram for $S = 0.06$, $C = 4\text{ }\mu\text{F}$ and the data of example 24.2

A large capacitor at start leads to higher starting torque due to symmetrization tendency at $s = 1$. At low slip values, however, there is too

much inverse torque (Figure 24.6) as the motor departs from symmetrization conditions.

A proper capacitor at rated load would lead again the symmetrization conditions (zero inverse current) and thus good performance is obtained.

This is how the dual-capacitor IM evolved. The ratio of the two capacitance is only 2/1 in our case but as power goes up in the KW range, this ratio may go up to 6/1.

The phasor diagram at $S = 0.06$ is shown on Figure 24.7.

As the equivalent circuit (Figure 24.5) and the current and torque components expressions (24.16)-(24.19) remain unchanged, the case of split-phase IM (without a capacitor, but with an additional resistance: $\Delta R_{sm} \neq 0$ by design or for real) may be treated the same way as the capacitor IM, except for the content of $\underline{Z}_a^m$.

24.4 SYMMETRIZATION CONDITIONS

By symmetrisation we mean zero inverse current component I_{m-} for a given slip. This requires by necessity a capacitance C dependent on slip. From (14.17) with $\underline{I}_{m-} = 0$, we obtain

$$(1 + j/a)\underline{Z}_+ + 2\underline{Z}_a^m = 0; \underline{Z}_+ = R_+ + jX_+ \quad (24.20)$$

with

$$\underline{Z}_a^m = -j\frac{1}{2\omega Ca^2} + \frac{1}{2}\left(\frac{R_{sa}}{a^2} - R_{sm}\right) + \frac{1}{2}j(X_{sa}/a^2 - X_{sm}) = R_a^m + jX_a^m \quad (24.21)$$

The direct total impedance $\underline{Z}_+$ may be written as

$$\underline{Z}_+ = Z_+ e^{j\phi_+}; \phi_+ = \tan^{-1}(X_+/R_+) \quad (24.22)$$

With (24.21-24.22), Equation (24.20) yields two symmetrization conditions,

$$R_+ - \frac{X_+}{a} = -2R_a^m = -\left(R_{sa}/a^2 - R_{sm}\right) \quad (24.23)$$

$$\frac{R_+}{a} + X_+ = +\frac{1}{\omega Ca^2} - \left(X_{sa}/a^2 - X_{sm}\right) \quad (24.24)$$

The two unknowns of Equations (24.23)-(24.24) are

- C and a for the capacitor IM
- R_{sa} and a for the split-phase IM

The eventual additional resistance in the auxiliary winding of the split-phase IM is lumped into R_{sa} .

For the permanent capacitor motor, in general, $\Delta R_{sm} = R_{sa}/a^2 - R_{sm} = 0$ and $\Delta X_{sm} = X_{sa}/a^2 - X_{sm} = 0$. So we obtain:

$$a = \frac{X_+}{R_+} = \tan \varphi_+ \quad (24.25)$$

$$X_c = \frac{1}{\omega C} = \frac{X_+}{\cos^2 \varphi_+} = Z_+ \cdot a \sqrt{a^2 + 1} \quad (24.26)$$

φ_+ is the direct component power factor angle.

A few remarks are in order

- The turn ratio a for symmetrization is equal to the tangent of the direct component of power factor angle. This angle varies with slip for a given motor.
- So, in dual capacitor-motor, only, it may be possible to match a and C both for start and running conditions when and only when the value of turn ratio a changes also from low to high speed running when the capacitor value is changed. This change of a means a dual section main or auxiliary winding.

For the split phase motor $C = \infty$ and $\Delta R_{sm} \neq 0$ and $\Delta X_{sm} \neq 0$ and thus from (24.23)-(24.24), even if

$$X_{sa} / a^2 < X_{sm} \quad (24.27)$$

we obtain

$$a = \frac{-R_+ \pm \sqrt{R_+^2 - 4(X_+ - X_{sm})X_{sa}}}{X_+ - X_{sm}} \quad (24.28)$$

It is now clear that as $X_+ > X_{sm}$, there is no a to symmetrize the split/phase IM, as expected.

Choosing a and R_{sa} should thus follow other optimisation criterion.

For the dual-capacitor motor IM in example 24.2, the value of a and C for symmetrization, according to (24.25)-(24.26) are

$$(a)_{s=1} = \frac{(X_+)_{s=1}}{(R_+)_{s=1}} = \frac{65.22}{57.5} = 1.134$$

$$(C)_{s=1} = \frac{1}{314 \cdot \left(\sqrt{57.5^2 + 65.22^2} \right) \times 1.134 \sqrt{1 + 1.134^2}} = 21.36 \times 10^{-6} \text{ F}$$

$$(a)_{s=0.06} = \frac{(X_+)_{s=0.06}}{(R_+)_{s=0.06}} = \frac{197.25}{139.4} = 1.415$$

$$(C)_{s=0.06} = \frac{1}{314 \left(\sqrt{139.25^2 + 197.25^2} \right) \cdot 1.415 \sqrt{1 + 1.415^2}} = 5.378 \cdot 10^{-6} \text{ F}$$

The actual values of the capacitor were 8 μF for starting ($S = 1$) and 4 μF for rated slip ($S = 0.06$). Also the ratio of equivalent turns ratio was $a = 1.73$.

It is now evident that with this value of a ($a = 1.73$) no capacitance C can symmetrize perfectly the machine for $S = 1$ or $S = 0.06$. Once the machine is made, finding a capacitor to symmetrize the machine is difficult. In practice, in general, coming close to symmetrization conditions for rated load is recommendable, while other criteria for a good design have to be considered..

24.5 STARTING TORQUE AND CURRENT INQUIRIES

For starting conditions ($S = 1$) $\underline{Z}_+ = \underline{Z}_- = \underline{Z}_{sc}$, so (24.16) and (24.17) become

$$\underline{I}_{m+} = \frac{\underline{V}_s}{2\underline{Z}_{sc}} \left[1 - \frac{j}{a} \frac{\underline{Z}_{sc}}{\underline{Z}_{sc} + 2\underline{Z}_a^m} \right] \quad (24.29)$$

$$\underline{I}_{m-} = \frac{\underline{V}_s}{2\underline{Z}_{sc}} \left[1 + \frac{j}{a} \frac{\underline{Z}_{sc}}{\underline{Z}_{sc} + 2\underline{Z}_a^m} \right] \quad (24.30)$$

The first components of $+$ - currents in (24.29)-(24.30) correspond to the case when the auxiliary winding is open ($|\underline{Z}_a^m| = \infty$)

We may now write

$$\underline{Z}_{sc} = Z_{sc} e^{j\varphi_{sc}}; \underline{Z}_a^m = Z_a^m e^{j\varphi_a}; K_a = \frac{2|\underline{Z}_a^m|}{|\underline{Z}_{sc}|} \quad (24.31)$$

This way (24.29) may be written as

$$\underline{I}_{m+} = \underline{I}_{sc} (1 + \alpha + j\beta) / 2 \quad (24.32)$$

$$\underline{I}_{m-} = \underline{I}_{sc} (1 - \alpha - j\beta) / 2 \quad (24.33)$$

with
$$\alpha + j\beta = \frac{-j}{a(1 + K_a \exp(j\gamma))} \quad (24.34)$$

with
$$\gamma = \varphi_{sc} - \varphi_a$$

Let us denote by T_{esc} the starting torque of two phase symmetrical IM.

$$T_{esc} = 2 \frac{P_1}{\omega_1} \cdot I_{sc}^2 \operatorname{Re}(\underline{Z}_{r+})_{S=1} \quad (24.35)$$

The p.u. torque at start t_{es} is

$$t_{es} = \frac{(T_e)_{s=1}}{T_{esc}} = \frac{|I_{m+}|^2 - |I_{m-}|^2}{|I_{sc}|^2} \quad (24.36)$$

Making use of (24.32)-(24.33) t_{es} becomes

$$t_{es} = \alpha = \frac{K_a \sin \gamma}{a(1 + K_a^2 + 2K_a \cos \gamma)} \quad (24.37)$$

$$\beta = -(1 + K_a \cos \gamma) / (1 + K_a^2 + 2K_a \cos \gamma) / a \quad (24.38)$$

A few comments are in order

- The relative starting torque depends essentially on the ratio K_a and on the angle γ .
- We should note that for the capacitor motor with windings using the same quantity of copper ($\Delta R_{sa} = 0$ $\Delta X_{sa} = 0$),

$$(K_a)_C = \frac{1}{\omega C a^2 |Z_{sc}|} \quad (24.39)$$

Also for this case $\varphi_a = -\pi/2$ and thus

$$(\gamma)_c = \varphi_{sc} + \pi/2 \quad (24.40)$$

- On the other hand, for the split phase capacitor motor with, say, $\Delta X_{sa} = 0$, $C = \infty$.

$$(K_a)_{\Delta R_{sa}} = \frac{\Delta R_{sa}}{|Z_{sc}|} = \frac{(R_{sa} - a^2 R_{sm})}{a^2 |Z_{sc}|} \quad (24.41)$$

Also $\varphi_a = 0$ and $\gamma = \varphi_{sc}$.

For the capacitor motor, with (24.39)-(24.40), the expression (24.37) becomes

$$t_{es} = \frac{\cos \varphi_{sc}}{a \left[a^2 Z_{sc} \cdot \omega_1 C + \frac{1}{\omega_1 C Z_{sc} a^2} - 2 \sin \varphi_{sc} \right]} \quad (24.42)$$

The maximum torque versus the turns ratio a is obtained for

$$a_K = \sqrt{\frac{\sin \varphi_{sc}}{Z_{sc} \omega C}} \quad (24.43)$$

$$(t_{es})_{\max} = \frac{\sqrt{Z_{sc} \omega_1 C \sin \varphi_{sc}}}{\cos \varphi_{sc}}$$

For the split phase motor (with ΔR_{sa} replacing $1/\omega C$) and $\gamma = \varphi_{sc}$, no such maximum starting torque occurs.

The maximum starting torque conditions are not enough for an optimum starting.

At least the starting current has to be also considered.

From (24.32)-(24.33)

$$I_m = (I_{m+} + I_{m-}) = I_{sc} \quad (24.44)$$

$$I_a = j \frac{(I_{m+} - I_{m-})}{a} = j \frac{(\alpha + j\beta)}{a} I_{sc} \quad (24.45)$$

The source current I_s is

$$I_s = \left[1 + \frac{(-\beta + j\alpha)}{a} \right] I_{sc} \quad (24.46)$$

$$\frac{I_s}{I_{sc}} = \sqrt{\left(1 - \frac{\beta}{a} \right)^2 + \frac{\alpha^2}{a^2}}$$

Finally for the capacitor motor

$$\frac{I_s}{I_{sc}} = \sqrt{\left[1 + \frac{t_{es}}{a} \frac{(\omega C Z_{sc} a^2 - \sin \varphi_{sc})}{\cos \varphi_{sc}} \right]^2 + \frac{t_{es}^2}{a^2}} \quad (24.47)$$

The p.u. starting torque and current expressions reflect the strong influence of: capacitance C , the shortcircuit impedance Z_{sc} , its phase angle, and the turns ratio a . As expected, a higher rotor resistance would lead to higher starting torque.

In general $1.5 < a < 2.0$ seems to correspond to most optimization criteria. From (24.42), it follows that in this case $Z_{sc}\omega C < 1$.

For the split phase induction motor with $\gamma = \varphi_{sc}$ from (24.38)

$$\beta = \frac{-(1 + K_a \cos \varphi_{sc})}{a(1 + K_a^2 + 2K_a \cos \varphi_{sc})} \quad (24.48)$$

with α (t_{es}) from (24.37)

$$\beta = \frac{-(1 + K_a \cos \varphi_{sc})}{K_a \sin \varphi_{sc}} \cdot t_{es} \quad (24.49)$$

With K_a from (24.41) the starting current (24.46) becomes

$$\frac{I_s}{I_{sc}} = \sqrt{\left[1 + \frac{t_{es}}{a} \frac{\left(\frac{|Z_{sc}|^2 a^2}{(R_{sa} - a^2 R_{sm})} + \cos \varphi_{sc} \right)}{\sin \varphi_{sc}} \right]^2} + \left(\frac{t_{es}}{a} \right)^2 \quad (24.50)$$

Expressions (24.47)-(24.50) are quite similar except for the role of φ_{sc} , the shortcircuit impedance phase angle. They allow to investigate the existence of a minimum I_s/I_{sc} for given starting torque for a certain turns ratio a_{ki} .

Especially for the capacitor start and split-phase IM the losses in the auxiliary winding are quite important as, in general, the design current density in this winding (which is to be turned off after start) is to be large to save costs.

Let us notice that for the capacitor motor, the maximum starting torque conditions (24.43) lead to a simplified expression of I_s/I_{sc} (24.47).

$$\left(\frac{I_s}{I_{sc}} \right)_{t_{es} \max} = \sqrt{1 + \left(\frac{(t_{es})_{\max}}{a_K} \right)^2} = \sqrt{1 + \frac{(Z_{sc} \omega C)^2}{\cos^2 \varphi_{sc}}} \quad (24.51)$$

For $Z_{sc} \omega C < 1$, required to secure $a_K > 1$, both the maximum starting torque (24.43) and the starting current are reduced.

For an existing motor, only the capacitance C may be changed. Then directly (24.43) and (24.47) may be put to work to investigate the capacitor influence on starting torque and on starting losses (squared starting current).

As expected there is a capacitor value that causes minimum starting losses (minimum of $(I_s/I_{sc})^2$) but that value does not also lead to maximum starting torque.

The above inquiries around starting torque and current show clearly that this issue is rather complex and application-dependent design optimization is required to reach a good practical solution.

24.6 TYPICAL MOTOR CHARACTERISTIC

To evaluate capacitor or split-phase motor steady-state performance, the general practice rests on a few widely recognized characteristics.

- Torque (T_e) versus speed (slip)
- Efficiency (η) versus speed
- Power factor ($\cos \varphi_s$) versus speed
- Source current (I_s) versus speed
- Main winding current (I_m) versus speed
- Auxiliary winding current (I_a) versus speed
- Capacitor voltage (V_c) versus speed
- Auxiliary winding voltage (V_a) versus speed

All these characteristics may be calculated via the circuit model in [Figure 24.2](#) and Equations (24.16-24.19) with (24.11)-(24.14), (24.44)-(24.45), for given motor parameters, capacitance C , and speed (slip).

Typical such characteristics for a 300 W, 2 pole, 50 Hz permanent capacitor motor are shown in [Figure 24.8](#).

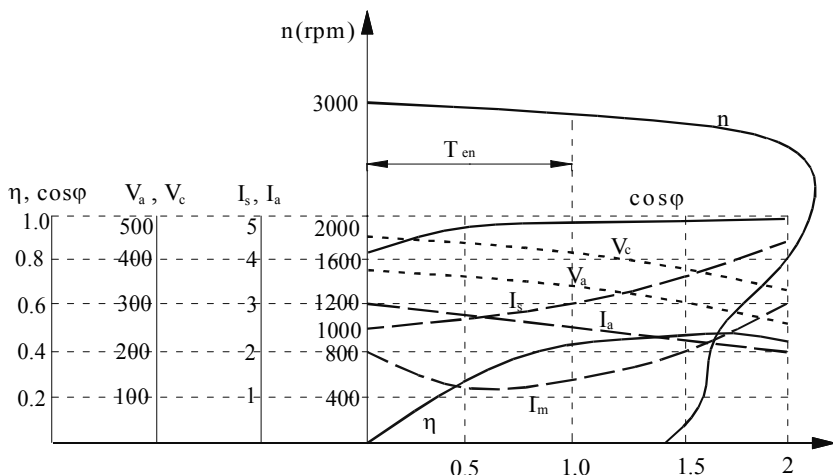


Figure 24.8 Typical steady-state characteristics of a 300W, 2 pole, 50Hz

The characteristics on [Figure 24.8](#) prompt remarks such as

- The torque is zero for a speed smaller than $n_1 = f_1/p_1 = 50$ rps (3000 rpm).
- The efficiency is only moderate as a rather large rotor resistance has been considered to secure good starting torque for moderate starting current.
- The power factor is high all over the torque (speed) range.
- The capacitor and auxiliary winding voltages (V_c , V_a) decrease with torque increase (above breakdown torque speed).
- Notice that data on [Figure 24.8](#) correspond to possible measurements above breakdown torque speed and not from the start. Only the torque-speed curve covers the entire speed range (from the start).
- From zero torque to rated torque (about 1 Nm), the source current increases only a little while I_a decreases and I_m has a minimum below rated torque.
- The characteristics on [Figure 24.8](#) are typical for a permanent capacitor IM where the designer has to trade efficiency for starting torque.

24.7 NON-ORTHOGONAL STATOR WINDINGS

It was shown that for the split-phase motor, with a more resistive auxiliary winding, there is no way to even come close to symmetry conditions with orthogonal windings.

This is where the non-orthogonal windings may come into play.

As the time phase angle shift $(\varphi_a - \varphi_m) < 90^\circ$, placing spatially the windings at an angle $\xi > 90^\circ$ intuitively leads to the idea that more forward travelling field might be produced.

Let us write the expressions of two such m.m.f.s

$$F_m(\theta_{es}, t) = F_{m1} \cos \theta_{es} \cdot \sin \omega_1 t \quad (24.52)$$

$$F_a(\theta_{es}, t) = F_{a1} \cos(\theta_{es} + \xi) \cdot \sin(\omega_1 t + (\varphi_a - \varphi_m)) \quad (24.53)$$

To the simple mathematics let us suppose that $F_{m1} = F_{a1}$ and that $(\xi > 90^\circ)$,

$$\xi + (\varphi_a - \varphi_m) = 180^\circ \quad (24.54)$$

$$\begin{aligned} F(\theta_{es}, t) &= F_m(\theta_{es}, t) + F_a(\theta_{es}, t) = \\ &= \frac{F_{m1}}{2} [\sin(\omega_1 t - \theta_{es}) + \sin(\omega_1 t - \theta_{es} + 180 - 2\xi)] \end{aligned} \quad (24.55)$$

For $\xi = 90^\circ$, the symmetry condition of capacitor motor is obtained. This corresponds to the optimum case.

For $\xi > 90^\circ$, still, a good direct travelling wave can be obtained. Its amplitude is affected by $\xi \neq 90^\circ$.

It is well understood that even with a capacitor motor, such an idea may be beneficial in the sense of saving some capacitance. But it has to be remembered that placing the two stator windings nonorthogonally means the using in common of all slots by the two phases.

For a split phase IM condition, (24.54) may not be easily met in practice as $\varphi_a - \varphi_m = 30^\circ - 35^\circ$ and $\xi = 145^\circ - 150^\circ$ is a rather large value.

To investigate quantitatively the potential of nonorthogonal windings, a transformation of coordinates (variables) is required (Figure 24.9)

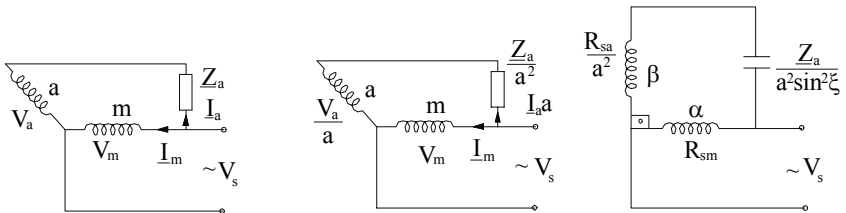


Figure 24.9 Evolutive equivalent schematics of nonorthogonal winding IM

The m & a nonorthogonal (real) windings are gradually transformed into orthogonal α - β windings. The conservation of m.m.f.s and apparent power leads to :

$$\begin{aligned}\underline{I}_\alpha &= \underline{I}_m + a\underline{I}_a \cos \xi \\ \underline{I}_\beta &= a\underline{I}_a \sin \xi \\ \underline{V}_\alpha &= \underline{V}_m \\ \underline{V}_\beta &= \frac{\underline{V}_a / a - \underline{V}_m \cos \xi}{\sin \xi}\end{aligned}\quad (24.56)$$

The total stator copper losses for the m & a and α & β model are

$$P_{m\&a} = R_{sm} I_m^2 + R_a I_a^2 \quad (24.57)$$

$$\begin{aligned}P_{\alpha\&\beta} &= R_{sm} I_\alpha^2 + \frac{R_a}{a^2} I_\beta^2 = \\ &= R_{sm} I_a^2 + R_a I_a^2 + R_{sm} \cdot 2I_a \cdot a \cdot I_m \cos \xi \cos(\varphi_m - \varphi_a)\end{aligned}\quad (24.58)$$

φ_m and φ_a are the phase angles of $\underline{I}_m$ and $\underline{I}_a$. It is now evident that only with $\varphi_m - \varphi_a = 90^\circ$ (the capacitor motor) the a and m and α and β are fully equivalent in terms of losses.

As long as ξ is close to 90° (up to 100 - 110°) and $\varphi_m - \varphi_a = 70^\circ$ - 80° , the two models are practically equivalent.

The additional impedance in the β circuit is obtained from

$$\underline{Z}_a I_a^2 = \underline{Z}_{a\beta} I_\beta^2 \quad (24.59)$$

With (24.56)-(24.59) becomes

$$\underline{Z}_{a\beta} = \frac{\underline{Z}_a}{a^2 \sin^2 \xi} \quad (24.60)$$

as shown on [Figure 24.9 c](#).

The relationship between the main and auxiliary terminal voltages is

$$\begin{aligned}\underline{V}_s &= \underline{V}_a + \underline{Z}_a \underline{I}_a = \underline{V}_{a0} + (R_{sa} + jX_{sa})\underline{I}_a + \underline{Z}_a \underline{I}_a \\ \underline{V}_s &= \underline{V}_m = \underline{V}_{m0} + \underline{I}_m (R_{sm} + jX_{sm})\end{aligned}\quad (24.61)$$

To consider the general case of stator windings when $R_{sa} / a^2 \neq R_{sm}$ and $X_{sa} / a^2 \neq X_{sm}$, we have introduced the total e.m.f.s $\underline{V}_{a0}$ and $\underline{V}_{m0}$ in (24.61).

Also, we introduce a coefficient m which is unity if $R_{sa} / a^2 \neq R_{sm}$ and $X_{sa} / a^2 \neq X_{sm}$ and zero, otherwise,

$$\begin{aligned}\underline{V}_s &= \underline{V}_{a0} + [\underline{Z}_a + (R_{sa} + jX_{sa})(1-m)]\underline{I}_a \\ \underline{V}_s &= \underline{V}_m = \underline{V}_{m0} + (R_{sm} + jX_{sm})(1-m)\underline{I}_m\end{aligned}\quad (24.62)$$

We now apply the symmetrical components method to V_α , V_β

$$\begin{aligned}\left|\frac{\underline{V}_+}{\underline{V}_-}\right| &= \frac{1}{2} \left| \frac{1-j}{1+j} \frac{\underline{V}_\alpha}{\underline{V}_\beta} \right| \\ \underline{V}_\alpha &= \underline{V}_+ + \underline{V}_- \\ \underline{V}_\beta &= j(\underline{V}_+ - \underline{V}_-)\end{aligned}\quad (24.63)$$

With (24.56), (24.60), (24.62), Equation (24.63) yields only two equations,

$$\begin{aligned}\underline{V}_s &= a[j(\underline{V}_+ - \underline{V}_-)\sin \xi + (\underline{V}_+ + \underline{V}_-)\cos \xi] - \frac{[Z_a + (R_{sa} + jX_{sa})(1-m)]}{ja \sin \xi} (\underline{I}_+ - \underline{I}_-) \\ \underline{V}_s &= \underline{V}_+ + \underline{V}_- + (R_{sm} + jX_{sm}) \left[\underline{I}_+ + \underline{I}_- - \frac{\cos \xi}{\sin \xi} j(\underline{I}_+ - \underline{I}_-) \right]\end{aligned}\quad (24.64)$$

The + - components equations are

$$\underline{V}_1 = \underline{Z}_+ \underline{I}_+; \underline{V}_2 = \underline{Z}_- \underline{I}_- \quad (24.65)$$

with

$$\underline{Z}_+ = (R_{sm} + jX_{sm})m + \frac{jX_{mm} \left(\frac{R_{rm}}{S} + jX_{rm} \right)}{\frac{R_{rm}}{S} + j(X_{mm} + X_{rm})} \quad (24.66)$$

$$\underline{Z}_- = (R_{sm} + jX_{sm})m + \frac{jX_{mm} \left(\frac{R_{rm}}{2-S} + jX_{rm} \right)}{\frac{R_{rm}}{2-S} + j(X_{mm} + X_{rm})} \quad (24.67)$$

Notice again the presence of factor m (with values of $m = 1$ and respectively 0 as explained above).

The electromagnetic torque components are

$$\begin{aligned}T_{e+} &= 2 \left[\text{Re}(\underline{V}_+ \underline{I}_+^*) - R_{sm} (\underline{I}_+)^2 m \right] \frac{p_l}{\omega_1} \\ T_{e-} &= 2 \left[\text{Re}(\underline{V}_- \underline{I}_-^*) - R_{sm} (\underline{I}_-)^2 m \right] \frac{p_l}{\omega_1} \\ T_e &= T_{e+} + T_{e-}\end{aligned}\quad (24.68)$$

Making use of (24.65), $\underline{I}_+$ and $\underline{I}_-$ are eliminated from (24.64) to yield

$$\underline{V}_+ = \frac{a_0}{a_1} \underline{V}_s \quad (24.69)$$

$$\underline{V}_- = - \frac{V_s - \underline{V}_+ \left[1 - (R_{sm} + jX_{sm})(1-m) \left(\frac{1 - j \frac{\cos \xi}{\sin \xi}}{\underline{Z}_+} \right) \right]}{1 + \frac{(R_{sm} + jX_{sm})}{\underline{Z}_-} \left(1 + j \frac{\cos \xi}{\sin \xi} \right)} \quad (24.70)$$

$$\underline{C}_2 = -aj \sin \xi + a \cos \xi + \frac{[\underline{Z}_a + (R_{sa} + jX_{sa})(1-m)]}{ja \underline{Z}_- \cdot \sin \xi} \quad (24.71)$$

$$\underline{a}_0 = 1 - \frac{\underline{C}_2}{1 + \frac{(R_{sm} + jX_{sm})}{\underline{Z}_-} (1-m) \left(1 + j \frac{\cos \xi}{\sin \xi} \right)} \quad (24.72)$$

$$\begin{aligned} a_1 = aj \sin \xi + a \cos \xi - \frac{\underline{Z}_a + (R_{sa} + jX_{sa})(1-m)}{ja \underline{Z}_+ \sin \xi} - \\ - (1 - \underline{a}_0) \left[1 + \frac{(R_{sm} + jX_{sm})}{\underline{Z}_+} (1-m) \left(1 - j \frac{\cos \xi}{\sin \xi} \right) \right] \end{aligned} \quad (24.73)$$

24.8 SYMMETRIZATION CONDITIONS FOR NON-ORTHOGONAL WINDINGS

Let us consider the case of $m = 1$ ($R_{sa} = a^2 R_{sm}$, $X_{sa} = a^2 X_{sm}$), which is most likely to be used for symmetrical conditions.

In essence

$$\underline{V}_- = 0 \quad (24.74)$$

$$\underline{V}_+ = \underline{V}_a = \underline{V}_m = \underline{V}_s \quad (24.75)$$

Further on, from (24.69)

$$\underline{a}_0 = \underline{a}_1 \quad (24.76)$$

With $m = 1$, from (24.71)-(24.73),

$$\underline{C}_2 = -aj \sin \xi + a \cos \xi + \frac{\underline{Z}_a}{ja \underline{Z}_- \cdot \sin \xi} \quad (24.77)$$

$$\underline{a}_0 = 1 - \underline{C}_2 \quad (24.78)$$

$$\underline{a}_1 = aj \sin \xi + a \cos \xi - \frac{\underline{Z}_a}{ja \underline{Z}_+ \cdot \sin \xi} + \underline{a}_0 - 1 \quad (24.79)$$

Finally, from (24.79) with (24.76)

$$\underline{Z}_a = a\underline{Z}_+ \sin \xi [j(a \cos \xi - 1) - a \sin \xi] \quad (24.80)$$

For the capacitor motor, with orthogonal windings ($\xi = \pi/2$)

$$\underline{Z}_a = -a\underline{Z}_+[a + j] = \frac{-j}{\omega C} \quad (24.81)$$

Condition (24.81) is identical to (24.26) derived for orthogonal windings.

For a capacitor motor, separating the real and imaginary parts of (24.80) yields:

$$a = \frac{X_+}{R_+ \sin \xi + X_+ \cos \xi} = \frac{\sin \varphi_+}{\sin(\varphi_+ + \xi)} \quad (24.82)$$

$$\frac{1}{\omega C} = a^2 X_+ \sin \xi + a R_+ \sin \xi (1 - a \cos \xi) \quad (24.83)$$

$$\text{with} \quad \tan \varphi_+ = X_+ / R_+ \quad (24.84)$$

Table 24.1. Symmetrization conditions for $\varphi_+ = 45^\circ$

ξ	60°	70°	80°	90°	100°	110°	120°
a	0.732	0.78	0.863	1.00	1.232	1.232	2.73
$\frac{1}{\omega C X_+}$	0.732	1.109	1.456	2.00	2.967	4.944	12.046!

The results on [Table 24.1](#) are fairly general.

When the windings displacement angle ξ increases from 60° to 120° (for the direct component power factor angle $\varphi_+ = 45^\circ$), both the turns ratio a and the capacitance reactance steadily increase.

That is to say, a smaller capacitance is required for say $\xi = 110^\circ$ than for $\xi = 90^\circ$ but, as expected, the capacitance voltage V_c and the auxiliary winding voltage are higher.

Example 24.3 Performance: orthogonal versus non-orthogonal windings

Let us consider a permanent capacitor IM with the parameters $R_{sm} = 32 \Omega$, $R_{sa} = 32 \Omega$, $R_{rm} = 35 \Omega$, $L_{rm} = 0.2 \text{ H}$, $L_{sm} = L_{sa} = 0.1 \text{ H}$, $L_{mm} = 2 \text{ H}$, $p_1 = 1$, $a = 1$, $V_s = 220 \text{ V}$, $\omega_1 = 314 \text{ rad/sec}$, $\xi = 90^\circ$ (or 110°); $C = 5 \mu\text{F}$, R_{core} (parallel resistance) = 10000Ω .

Let us find the steady-state performance versus slip for orthogonal ($\xi = 90^\circ$) and non-orthogonal ($\xi = 110^\circ$) placement of windings.

A C++ code was written to solve this problem and the results are shown on [Figures. 24.10-24.11](#).

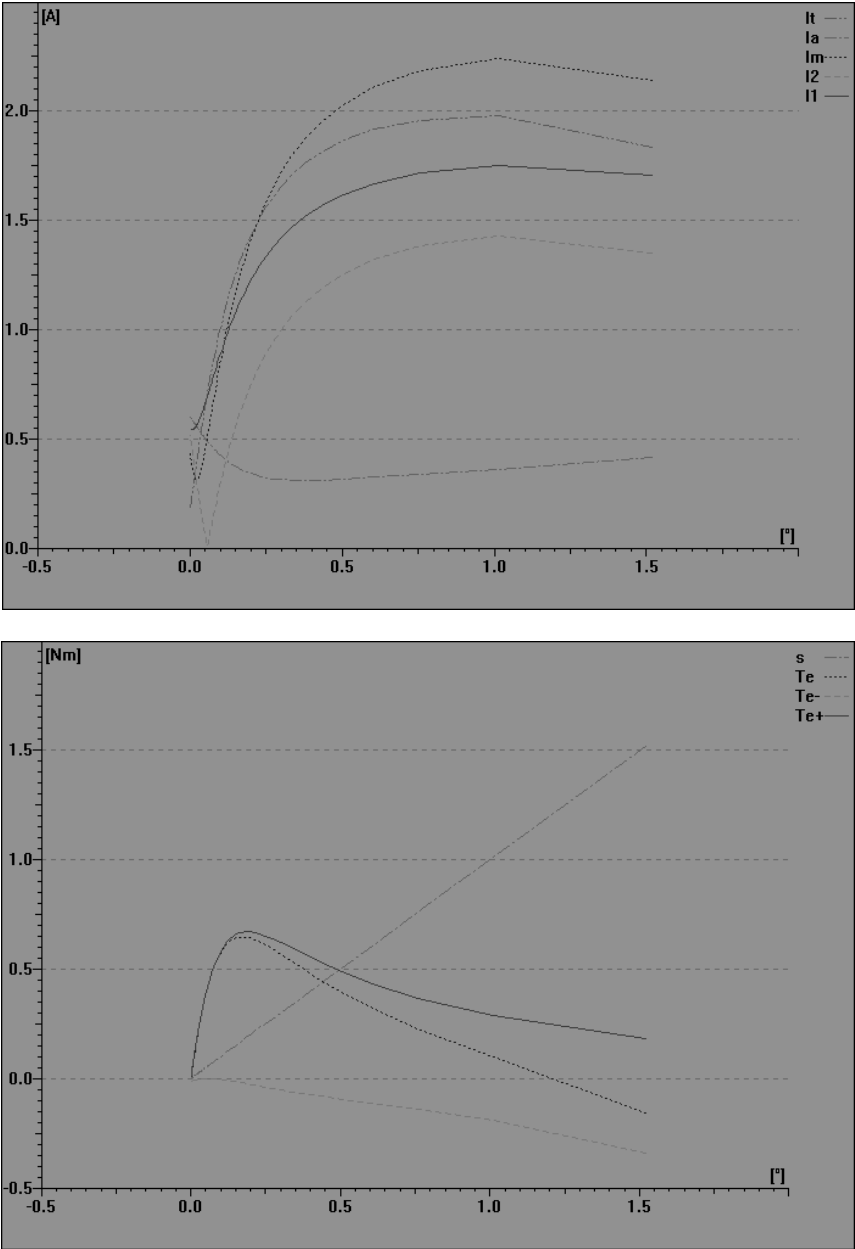


Figure 24.10 Performance with orthogonal windings ($\xi = 90^\circ$) (continued)

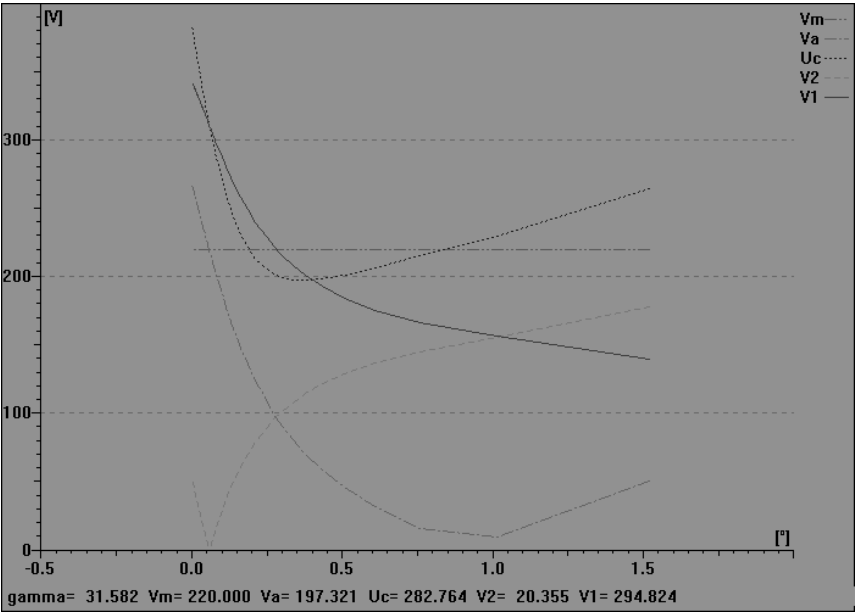
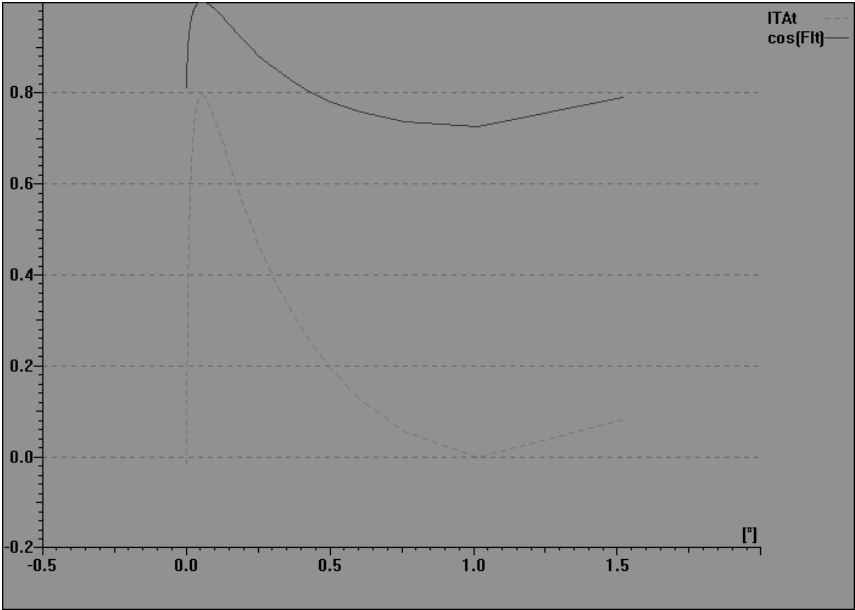


Figure 24.10 (continued)

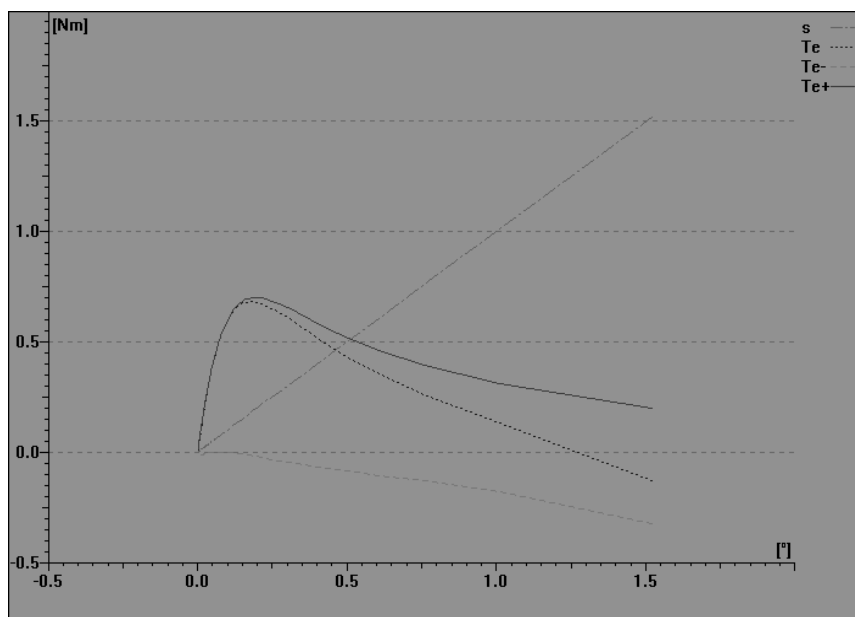
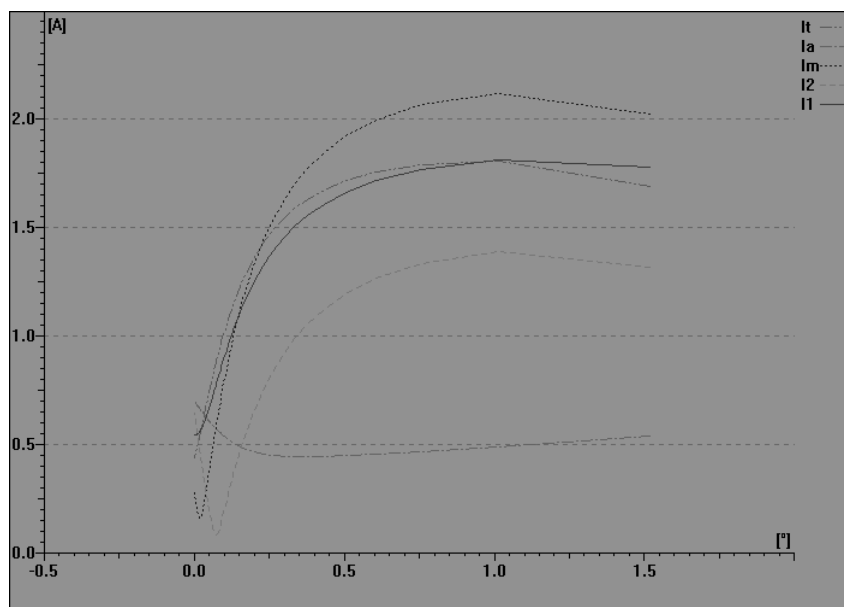


Figure 24.11 Performance with non-orthogonal windings ($\xi = 110^\circ$) (continued)

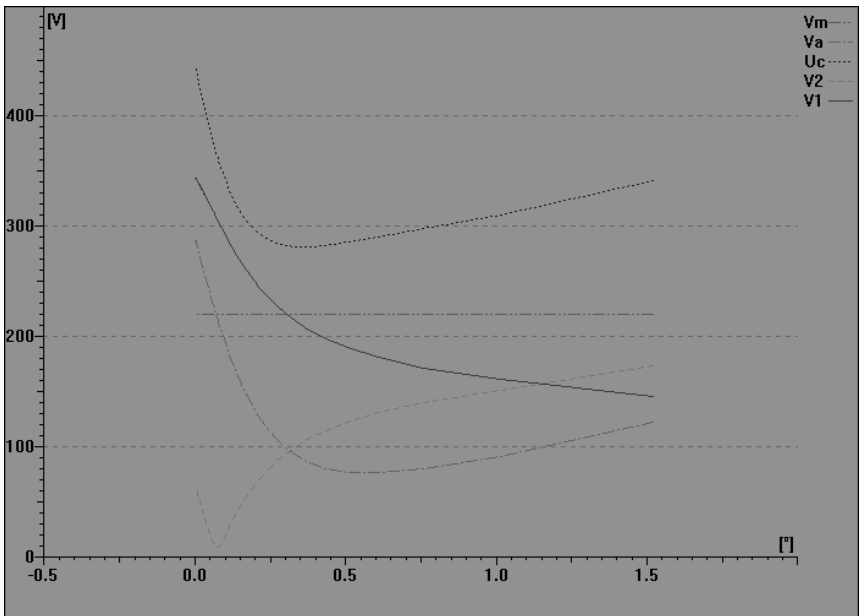
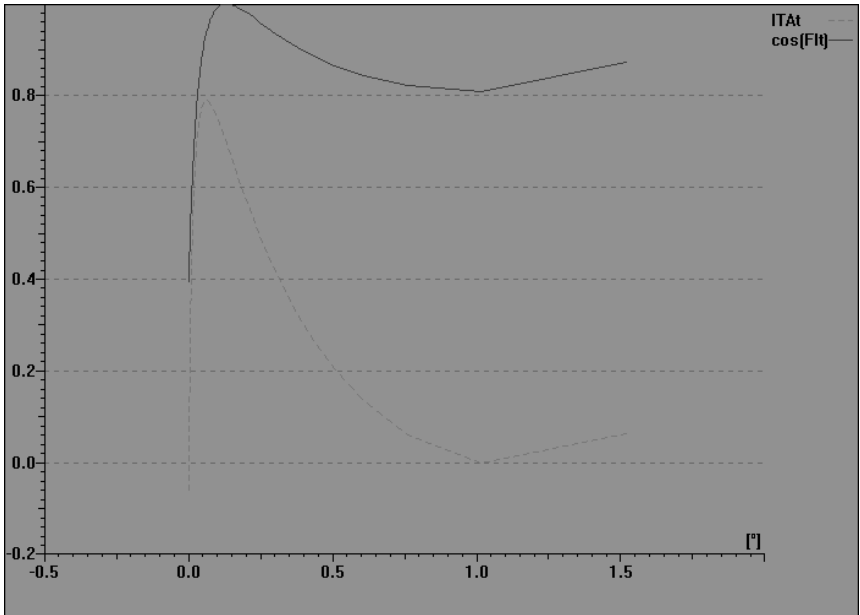


Figure 24.11 (continued)

The performance illustrated on [Figures 24.10-24.11](#) leads to remarks such as:

- The torque/speed curves are close to each other but more starting torque is obtained for the non-orthogonal windings motor.

- The symmetrization (zero I. and V.) slip is larger for the non-orthogonal winding motor.
- The power factor of the non-orthogonal winding motor is larger at 20% slip but smaller at slip values below 10%.
- The efficiency curves are very close.
- The capacitor voltage $\underline{V}_c$ is larger for the non-orthogonal winding motor.

It is in general believed that non-orthogonal windings make a more notable difference in split/phase rather than in capacitor IMs.

The mechanical characteristics ($T_e(S)$) for two different capacitors ($C = 5 \times 10^{-6} \text{ F}$ and $C_a = 20 \times 10^{-6} \text{ F}$) are shown on Figure 24.12 for orthogonal windings.

As expected, the larger capacitor produces larger torque at start but it is not adequate at low slip values because it produces a too large inverse torque component.

The same theory can be used to investigate various designs in the pursuit of chosen optimization criterion such as minimum auxiliary winding loss during starting, etc.

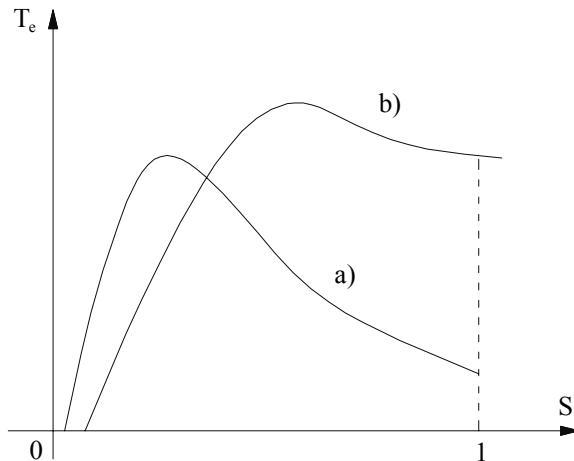


Figure 24.12 Torque/slip curves
a.) $C = 5 \times 10^{-6} \text{ F}$; b.) $C_a = 20 \times 10^{-6} \text{ F}$

The split phase motor may be investigated as a particular case of capacitor motor with either $\underline{Z}_a = 0$ and increased R_{sa} value or with the original (real) R_{sa} and $\underline{Z}_a = R_a$. Above a certain slip, the auxiliary phase is turned off which means $|\underline{Z}_a| \approx \infty$.

A general circuit theory to handle 2 or 3 stator windings located at random angles has been developed in [5,6].

24.9 M.M.F. SPACE HARMONIC PARASITIC TORQUES

So far only the fundamental of m.m.f. space distribution along rotor periphery has been considered for the steady-state performance assessment.

In reality, the placement of stator conductors in slots leads to space m.m.f. harmonics. The main space harmonic is of the third order. Notice that the current is still considered sinusoidal in time.

From the theory of three phase IMs, we remember that the slip S_v for the v^{th} space harmonics is

$$S_v = 1 \mp v(1 - s) \quad (24.85)$$

For a single phase IM (the auxiliary phase is open) the space harmonic m.m.f.s are stationary, so they decompose into direct and inverse travelling components for each harmonic order.

Consequently, the equivalent circuit in Figure 24.2 may be adjusted by adding rotor equivalent circuits for each harmonic (both direct and inverse waves)-Figure 24.13.

Once the space harmonic parameters X_{rmv} , X_{mmv} are known, the equivalent circuit on Figure 24.13 allows for the computation of the additional (parasitic) asynchronous torques.

In the case when the reaction of rotor current may be neglected (say for the third harmonic), their equivalent circuits on Figure 24.14 are reduced to the presence of their magnetization reactances X_{mmv} . These reactances are not difficult to calculate as they correspond to the differential leakage reactances already defined in detail in Chapter 6, paragraph 6.1.

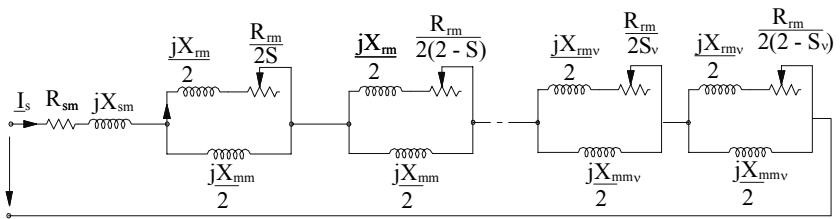


Figure 24.13 Equivalent circuit of single phase IM with m.m.f. space harmonics included

The presence of slot openings and of magnetic saturation enhances the influence of m.m.f. space harmonics on additional losses in the rotor bars and on parasitic torques.

Too large a capacitor would trigger deep magnetic saturation which produces a large 3rd order space harmonic. Consequently, a deep saddle in the torque/slip curve, around 33% of ideal no-load speed, occurs (Figure 24.14).

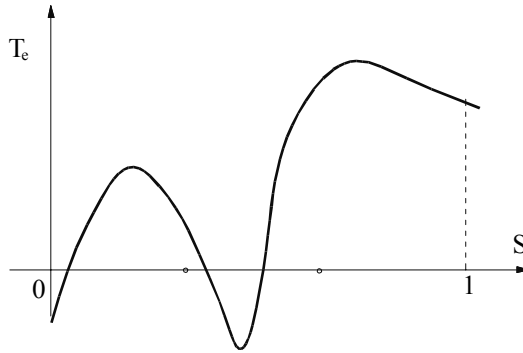


Figure 24.14 Grossly oversized capacitor IM torque-speed curve

Besides asynchronous parasitic torques, synchronous parasitic torques may occur as they do for three phase IMs. The most important speeds at which such torques may occur are zero and $2f_1/N_r$ (N_r -rotor slots).

When observing the conditions

$$N_s \neq N_r \quad (24.86)$$

$$|N_r - N_s| \neq 2p_1 \quad (24.87)$$

the synchronous torques are notably reduced (Figure 24.15).

The reduction of parasitic synchronous torques, especially at zero speed, is required to secure a safe start and reduce vibration, noise, and additional losses.

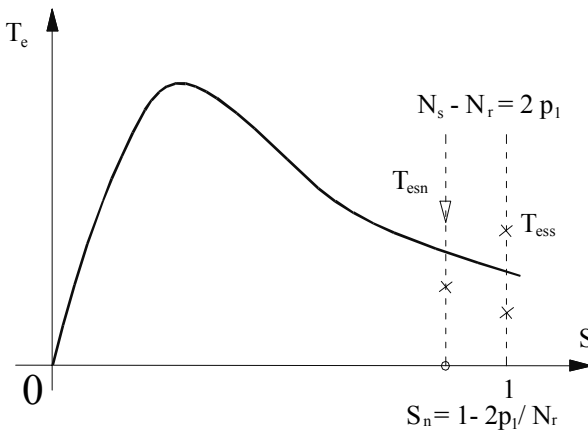


Figure 24.15 Synchronous parasitic torques

24.10 TORQUE PULSATIONS

The speed of the elliptical travelling field, present in the airgap of single phase or capacitor IMs, varies in time in contrast to the symmetrical IM where the speed of the circular field is constant.

As expected, torque pulsations occur.

The equivalent circuits for steady state are helpful to calculate only the time average torque.

Besides the fundamental m.m.f. produced torque pulsations, the space harmonics cause similar torque pulsations.

The d-q models for transients, covered in the next chapter, will deal with the computation of torque pulsations during steady state also.

Notable radial uncompensated forces occur, as in three phase IMs, unless the condition (24.88) is met

$$N_s \pm p_1 \neq N_r \quad (24.88)$$

24.11 INTER-BAR ROTOR CURRENTS

In most single phase IMs, the rotor cage bars are not insulated and thus rotor currents occur between bars through the rotor iron core. Their influence is more important in two pole IMs where they span over half the periphery for half a period (Figure 24.16).

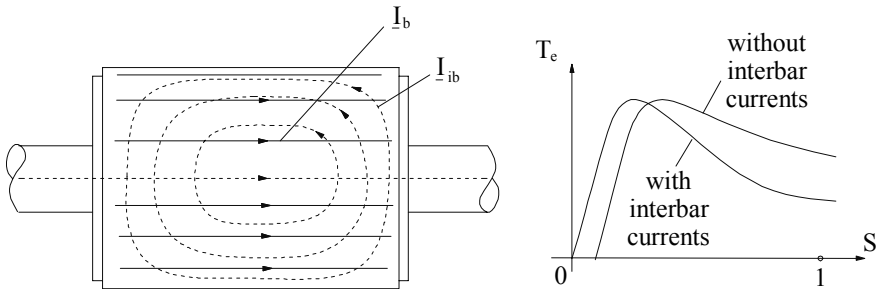


Figure 24.16 Interbar rotor currents and their torque ($2p_1 = 2$ poles)

The interbar current (I_{ib}) m.m.f. axis is displaced by 90° (electrical) with respect to the rotor bar current (I_b) m.m.f.

This is why the interbar currents are called transverse currents.

The interbar currents depend on the rotor skewing, besides the contact resistance between rotor bars and rotor iron and on rotor slip frequency.

The torque/speed curve may be influenced notably (as in Figure 24.16) by 2 pole IMs.

The slot harmonics may further increase the interbar currents as the frequency of their produced rotor currents is much higher than for the fundamental ($f_2 = Sf_1$).

Special thermal treatments of the rotor may increase the bar-core rotor resistance to reduce the interbar currents and thus increase the torque for large slip values.

The influence of rotor skewing and rotor stack length/pole ratio on interbar current losses are as for the three phase IMs (see Chapter 11, paragraph 11.12). Larger stacks tend to accentuate the interbar current effects. Also a large number of rotor slots ($N_r \gg N_s$) tend to lead to large interbar currents.

24.12 VOLTAGE HARMONICS EFFECTS

Voltage time harmonics occur in a single phase IM either when the motor is fed through a static power converter or when the power source is polluted with voltage (and current) time harmonics from other static power converters acting alone.

Both odd and even time harmonics may occur if there are imbalances within the static power converters.

High efficiency low rotor resistance single phase capacitor IMs are more sensitive to voltage time harmonics than low efficiency single phase induction motors with open auxiliary phase in running conditions.

For some voltage time harmonics, frequency resonance conditions may be met in the capacitor IM and thus high time harmonics currents occur. They in turn lead to marked efficiency reduction.

As the frequency of time voltage harmonics increases, so does the skin effect. Consequently, the leakage inductances decrease and the rotor resistance increases markedly. A kind of leakage flux “saturation” occurs. Moreover, main flux path saturation also occurs.

So the investigation of voltage time harmonics influence presupposes known, variable, parameters in the single phase capacitor IM.

Such parameters may be calculated or measured. Correctly measured parameters are the safest way, given the nonlinearities introduced by magnetic saturation and skin effect.

Once these parameters are known, for each voltage time harmonic, equivalent forward and backward circuits for the main and auxiliary winding may be defined.

For a capacitor IM, the equivalent circuit of [Figure 24.5](#) may be simply generalized for the time harmonic of order h ([Figure 24.16](#)).

The method of solution is similar to that described in paragraph 24.3.

The main path flux saturation, may be considered to be related only to the forward (+) component of the fundamental. X_{mm} in the upper part of [Figure 24.5](#) depends on the magnetization current I_{mm+} . Results obtained in a similar way (concerning harmonic losses), for distinct harmonic orders, are shown in [Figure 24.18](#), for a 1.5 kW, 12.5 μ F capacitor single phase induction motor. [9]

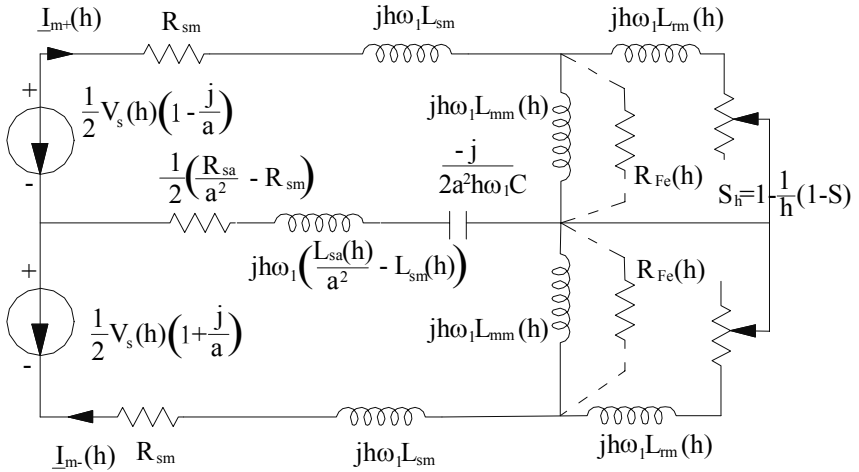


Figure 24.17 Time harmonics voltage equivalent circuit of capacitor IMs

Resonance conditions are met somewhere between the 11th and 13th voltage time harmonics. An efficiency reduction of 5% has been measured for this situation [9].

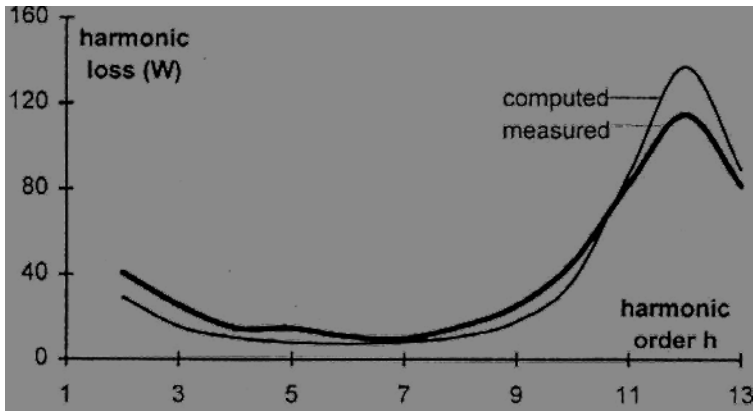


Figure 24.18 Harmonic losses at rated load and 10% time voltage harmonics

It is generally argued that the computation of total losses-including stray load losses, etc., is very difficult to perform as visible in Figure 24.18. Experimental investigation is imperative, especially when the power decreases and the number of poles increases and severe interbar rotor currents may occur.

The neglect of space harmonics in the equivalent circuit on Figure 24.5 and 24.17 makes the latter particularly adequate only from zero to breakdown torque slip.

24.13 THE DOUBLY TAPPED WINDING CAPACITOR IM

As already alluded to in Chapter 23 tapped windings are used with capacitor multispeed IMs or with dual-capacitor IMs (Figure 23.4) or with split-phase capacitor motors (Figure 23.8).

In the L or T connections (Figure 23.6), the tapping occurs in the main winding. For dual-capacitor or split phase capacitor IMs (Figures 23.4 and 23.8), the tapping is placed in the auxiliary winding. Here we are presenting a fairly general case (Figure 24.19).

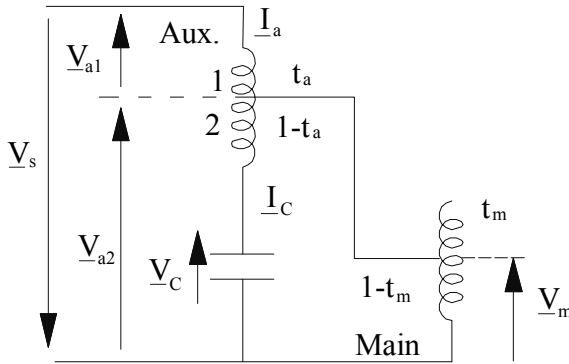


Figure 24.19 Doubly tapped winding capacitor IM

The two taps are t_a , for the auxiliary winding, and t_m for the main winding. In practice, only one tap is used at any time.

When only the tapping on the main winding is used $t_a = 0$ and $0 < t_m < 0.5$ in general. On the contrary, when the tapping is done in the auxiliary winding $0 < t_a < 0.5$ with $t_m = 0$. [8]

As it is evident from Figure 24.19, the main winding is present with a single circuit even for $t_m \neq 0$ (main winding tapping). Meanwhile, the tapped auxiliary winding is present with both sections aux 1 and aux 2.

Consequently, between the two sections there will be a mutual inductance.

Denoting by X the total main self inductance of the auxiliary winding, $X_1 = t_a^2 X$ represents aux 1 and $X_2 = (1 - t_a^2) X$ refers to auxiliary 2. The mutual inductance X_{12} is evidently $X_{12} = X t_a (1 - t_a)$. The coupling between the two auxiliary winding sections may be represented as in Figure 24.20.

Besides the main flux inductances, the leakage inductance components and the resistances are added. What is missing in Figure 24.20 is rotor and main windings e.m.f.s in the auxiliary winding sections and in the main winding.

As part of main winding is open (Figure 24.19) when tapping is applied, the equivalent circuit is much simpler (no coupling occurs).

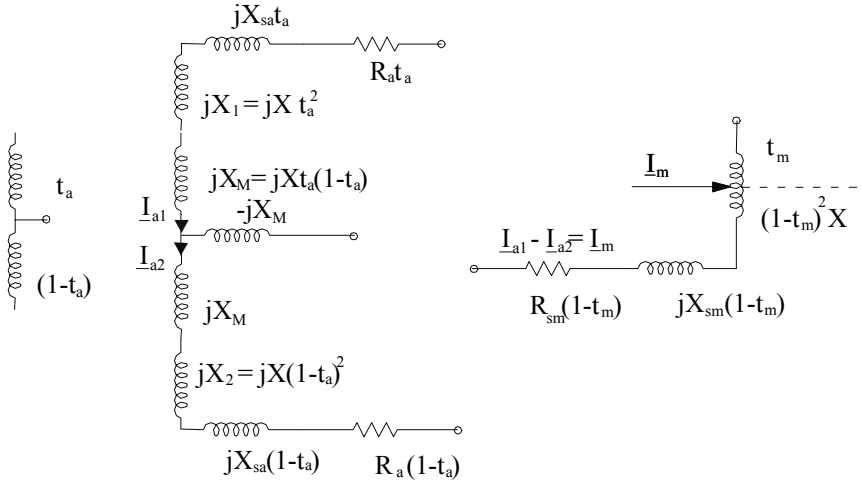


Figure 24.20 The tapped auxiliary a) and main b) winding equivalent (noncoupled) circuit

Let us denote as forward and backward $\underline{Z}_f, \underline{Z}_b$, the rotor equivalent circuits

$$\underline{Z}_f = \frac{1}{2} \frac{jX_{mm}(R_{rm}/S + jX_{rm})}{R_{rm}/S + j(X_{mm} + X_{rm})} \quad (24.89)$$

$$\underline{Z}_b = \frac{1}{2} \frac{jX_{mm}(R_{rm}/(2-S) + jX_{rm})}{R_{rm}/(2-S) + j(X_{mm} + X_{rm})} \quad (24.90)$$

We may now apply the revolving field theory (Morrill's theory) to this general case, observing that each of the two sections of the auxiliary winding and the main winding are producing self induced forward and backward e.m.f.s $\underline{E}_{fa1}, \underline{E}_{ba1}, \underline{E}_{fa2}, \underline{E}_{ba2}, \underline{E}_{fm}, \underline{E}_{bm}$. The main winding tapping changes the actual turn ratio to $a/(1-t_m)$; also $\underline{Z}_f$ and $\underline{Z}_b$ to $\underline{Z}_f (1-t_m)^2$ and $\underline{Z}_b (1-t_m)^2$.

$$\begin{aligned} \underline{E}_{fa1} &= t_a^2 a^2 \cdot \underline{Z}_f I_{a1} \\ \underline{E}_{ba1} &= t_a^2 a^2 \cdot \underline{Z}_b I_{a1} \\ \underline{E}_{fa2} &= (1-t_a)^2 a^2 \cdot \underline{Z}_f I_{a2} \\ \underline{E}_{ba2} &= (1-t_a)^2 a^2 \cdot \underline{Z}_b I_{a2} \\ \underline{E}_{fm} &= \underline{Z}_f \cdot (1-t_m)^2 I_m \\ \underline{E}_{bm} &= \underline{Z}_b \cdot (1-t_m)^2 I_m \end{aligned} \quad (24.91)$$

with:

$$\begin{aligned}
 \underline{Z}_{11} &= (1-t_m)(R_{sm} + jX_{sm}) + (\underline{Z}_f + \underline{Z}_b)(1-t_m)^2 + t_a(R_{sa} + jX_{sa}) + \\
 &\quad + t_a^2 a^2 (\underline{Z}_f + \underline{Z}_b) \\
 \underline{Z}_{22} &= -(1-t_m)(R_{sm} + jX_{sm}) - (\underline{Z}_f + \underline{Z}_b)(1-t_m)^2 - \\
 &\quad - (1-t_a)(R_{sa} + jX_{sa}) - (1-t_a)^2 a^2 (\underline{Z}_f + \underline{Z}_b) - \underline{Z}_c \\
 \underline{Z}_{12} &= -ja(1-t_m)(\underline{Z}_f - \underline{Z}_b) - (1-t_m)(R_{sm} + jX_{sm}) - (\underline{Z}_f + \underline{Z}_b)(1-t_m)^2 + \\
 &\quad + jt_a(1-t_a)a^2 X_{mm} \\
 \underline{Z}_{21} &= -ja(1-t_m)(\underline{Z}_f - \underline{Z}_b) + (1-t_m)(R_{sm} + jX_{sm}) + (\underline{Z}_f + \underline{Z}_b)(1-t_m)^2 - \\
 &\quad - jt_a(1-t_a)a^2 X_{mm}
 \end{aligned}$$

For no tapping in the main winding ($t_m = 0$), the results in Ref. 8 are obtained. Also with no tapping at all ($t_m = t_a = 0$), (24.91-24.94) degenerate into the results obtained for the capacitor motor ((24.16)-(24.17) with (24.10); $I_{a2} = I_a$, $V_m = V_{a2} = V_s$).

The equivalent circuit on Figure 24.20, for no tapping ($t_m = t_a = 0$) degenerates into the standard revolving theory circuit of the capacitor IM (Figure 24.22).

The torque may be calculated from the power balance

$$\begin{aligned}
 T_e &= [\text{Re}(\underline{V}_m \underline{I}_m^*) - R_{sm}(1-t_m)I_m^2 + \text{Re}(\underline{V}_{a1} \underline{I}_{a1}^*) + \\
 &\quad \text{Re}(\underline{V}_{a2} \underline{I}_{a2}^*) - t_a R_{sa} I_{a1}^2 - (1-t_a)R_{sa} I_{a2}^2] \cdot p_l / \omega_l
 \end{aligned} \quad (24.94)$$

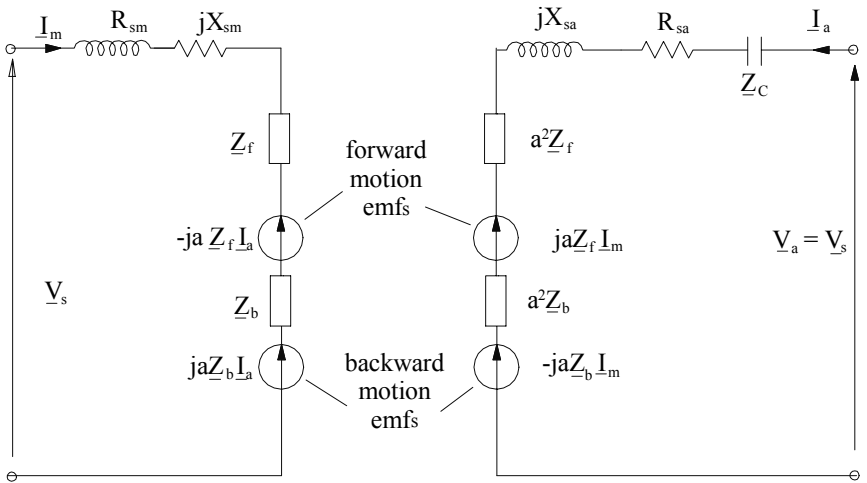


Figure 24.22 The revolving theory equivalent circuit of the capacitor motor

Symmetrization conditions

The general symmetrization may be determined by zeroing the backward (inverse) current components. However, based on geometrical properties, it is easy to see that, for orthogonal windings, orthogonal voltages in the main and auxiliary winding sections, with same power factor angle in both windings, correspond to symmetry conditions (Figure 24.23).

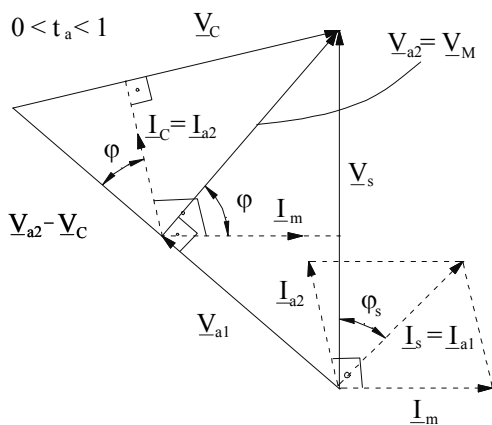


Figure 24.23 The phasor diagram for symmetrization conditions of the generally tapped winding capacitor IM

The two currents $\underline{I}_{a2}$, $\underline{I}_m$ are 90° phase shifted as the voltages are. On the other hand the current $\underline{I}_{a1}$ in the first section of the auxiliary winding a_1 is highly reactive and it serves basically for voltage reduction. This way some speed reduction is produced.

When no tapping is performed $t_a = 0$, $\underline{V}_{a1} = 0$, and thus $\underline{V}_{a2} = \underline{V}_m = \underline{V}_s$ and simpler symmetrization conditions are met (Figure 24.24).

This case corresponds to highest $\underline{V}_{a2} - \underline{V}_c$ voltage; that is, for highest auxiliary active winding voltage. Consequently, this is the highest speed situation expected.

Note on magnetic saturation and steady-state losses

As mentioned previously (in Chapter 23), magnetic saturation produces a flattening of the airgap flux density distribution. That is, a third harmonic e.m.f. and thus a third harmonic current may occur. The elliptic character of the magnetic field in the airgap makes the accounting of magnetic saturation even more complicated. Analytical solutions to this problem tend to use an equivalent sinusoidal current to handle magnetic saturation. It seems however that only FEM could offer a rather complete solution to magnetic saturation.

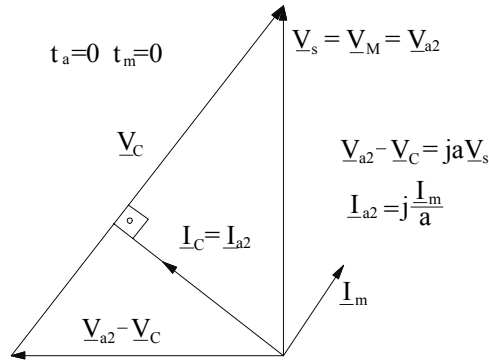


Figure 24.24 The phasor diagram for symmetrization conditions with no tapping

This appears even more so if we need to calculate the iron losses. On the other hand, experiments could be useful to assess magnetic saturation and fundamental and additional losses both in the core and in the rotor cage or losses caused by the interbar rotor iron currents. More on these complicated issues in the next two chapters.

24.14 SUMMARY

- The steady state performance of single phase IM is traditionally approached by the revolving field theory or the + – symmetrical components model. In essence, the voltages and currents in the two phases are decomposed into + and – components for which the slip is S and, respectively $2-S$. Superposition is applied. Consequently, magnetic saturation cannot be accounted for unless the backward component is considered not to contribute to the level of magnetic saturation in the machine.
- On the other hand, the d-q (or crossfield) model (in stator coordinates), with $d/dt = j\omega_1$ may be used for steady state. In this case, keeping the current sinusoidal, magnetic saturation may be iteratively considered. This model has gained some popularity with T-L or generally tapped winding multispeed capacitor IMs.
- The revolving theory (or + – or fb) model leads, for the single phase IM with open auxiliary winding, to a very intuitive equivalent circuit composed of + and – equivalent circuits in series. The torque is made of + and – components and is zero at zero speed ($S = 1$) as T_+ and T_- are equal to each other. At zero slip ($S = 0$) $T_+ = 0$ but $T_- \neq 0$ (and negative). Also at $S = 0$, with the auxiliary phase open, half of the rotor cage losses are covered electrically from stator and half mechanically from the driving motor.
- The split-phase and capacitor IM travelling field (+ –) model yields a special equivalent circuit with two unknowns I_{m+} , I_{m-} (the main winding current components). The symmetrization conditions are met

when $I_{m-} = 0$. As the speed increases above breakdown torque speed so do the auxiliary winding and capacitor voltages for the permanent capacitor IM. Capacitor IMs have a good power factor and a moderate efficiency in general.

- Besides the configuration with orthogonal windings, it is feasible to use non-orthogonal windings especially with split-phase IMs in order to increase the output. It has been shown that symmetrization conditions correspond to

$$\xi + (\varphi_a - \varphi_m) = 180^\circ$$

where ξ is the electrical spatial angle of the two windings and $\varphi_a - \varphi_m$ is the time phase angle shift between I_a and I_m .

The above conditions cannot be met in split-phase IMs but they may be reached in capacitor motors. Apparently less capacitance is needed for $\xi > 90^\circ$ and slightly more starting torque may be produced this way. However, building non-orthogonal windings means implicitly to use most or all slots, for both main and auxiliary windings.

- The split phase IM may be considered as a particular case of the capacitor motor with $Z_a = R_{aux}$. Above a certain speed the auxiliary phase is open ($R_{aux} = \infty$).
- Space harmonics occur due to the winding placement in slots. The third harmonic and the first slot harmonic are especially important. The equivalent circuit may be augmented with the space harmonics contribution (Figure 24.13). Asynchronous, parasitic, forward and backward (+, -) torques occur.
- For the permanent capacitor IM, the symmetrization conditions ($I_{m-} = 0$) yield rather simple expressions for the turns ratio a between auxiliary and main winding and the capacitor reactance X_c

$$a = \tan \varphi_+; X_c = \frac{X_+}{\cos^2 \varphi_+}$$

where φ_+ is the power factor angle for the + component and X_+ is the + component equivalent reactance.

- Starting torque is a major performance index. For a capacitor motor, it is obtained for a turn ratio a_K

$$a_K = \sqrt{\frac{\sin \varphi_{sc}}{Z_{sc} \omega C}}$$

Unfortunately, the maximum starting torque condition does not provide, in general, for overall optimum performance.

As for an existing motor only the capacitance C may be changed, care must be exercised when the turns ratio a is chosen.

- Starting current is heavily dependent on the turns ratio a and the capacitor C . A large capacitor is needed to produce sufficient starting torque but starting current limitation must also be observed.
- In general, for good starting performance of a given motor, a much larger capacitor C_s is needed in comparison with the condition of good running performance C ($C_s \gg C$).
- Torque, main and auxiliary currents, I_m , I_a , auxiliary winding voltage V_a , capacitor voltage V_c , efficiency and power factor versus speed, constitute standard steady state performance characteristics.
- The third harmonics may produce, with a small number of slots/pole, a large deep in the torque/speed curve around 33% of ideal no load speed ($n_1 = f_1/p_1$). Magnetic saturation tends to accentuate this phenomenon.
- When $N_s = N_r$ synchronous parasitic torques occur at zero speed. For $N_s - N_r = 2p_1$, such torques occur at $S = 1 - 2p_1/N_r$. So, in general

$$N_s \neq N_r; N_s - N_r \neq 2p_1$$

to eliminate some very important synchronous parasitic torques.

- To avoid notable uncompensated radial forces $N_s \pm p_1 \neq N_r$, as for three phase IMs.
- As the rotor bars are not insulated from rotor core, interbar currents occur. In small ($2p_1 = 2$) motors, they notably influence the torque/speed curves acting as an increased rotor resistance. That is, the torque is reduced at low slips and increased at high slip values when the bar-core contact resistance is increased.
- Voltage time harmonics originating from static power converters may induce notable additional losses in high efficiency capacitor IMs, especially around electrical resonance conditions, when the efficiency may be reduced to 3-5%. Such resonance voltage time harmonics have to be filtered out.
- Winding tapping is traditionally used to reduce speed for various applications. A fairly general revolving field model for dual winding tapping was developed. Symmetrisation conditions are illustrated (Figures 24.23, 24.24). They show the orthogonality of main and auxiliary windings voltages and currents for orthogonally placed stator windings.
- Magnetic saturation is approachable through the + (forward) magnetization inductance functional $L_{mm}(I_{mm+})$, but it seems to lead to large errors, especially when the backward current component is notable. [10] A more complete treatment of saturation is performed in Chapter 25 on transients.

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